TEACHERS' ATTENTION TO AND FLEXIBILITY WITH REFERENT UNITS

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In this study, we explored teachers' attention to and flexibility with referent units as well as how teachers' understanding of referent units is related to their performance on other fraction concepts and their professional background. By using data collected from 246 U.S. mathematics teachers in Grades 3–7 where fractions are taught, we found that teachers' attention to and flexibility with referent units were moderately related. Whereas some teachers' professional background variables could explain their flexibility with referent units, none of the variables was linked to their attention to referent units. Furthermore, both teachers' attention to and flexibility with referent units seemed to be associated with their performance on other fraction concepts.

Keywords: Rational Numbers, Teacher Knowledge

Fractions are critical content in the upper elementary and middle grades curriculum (e.g., Common Core State Standard Initiatives [CCSSI], 2010). Despite teachers' computing well on fraction arithmetic, they usually struggle with understanding fractions conceptually (e.g., Izsák, 2008). For instance, teachers can confuse problem situations asking for division by a fraction with those asking for multiplication by a fraction (e.g., Ma, 1999) or overgeneralize rules for whole numbers to fractions such as division makes numbers smaller (Jansen & Hohensee, 2016).

Several scholars have argued that such difficulties with understanding fractions might be related to the whole number bias (e.g., Vamvakoussi, Christou, & Vosniadou, 2018), whereas others have argued that not understanding number magnitude may be the underlying reason (e.g., Siegler, 2016). Scholars in mathematics education have also brought up referent units (RU), which are critical, yet overlooked, for having a conceptual understanding of fractions (e.g., Izsák, Orrill, Cohen, & Brown, 2010). Empirical work has provided support for the importance of RU (e.g., Izsák, Jacobson, and Bradshaw, 2019). For example, Izsák et al. (2010) analyzed 201 U.S. middle grades teachers' responses to a set of items and found two classes that distinguish the teachers based on their understanding of RU. In a recent study that analyzed 990 U.S. middle grades teachers' responses to a multiple-choice assessment, Izsák et al. (2019) found that teachers who were proficient in RU tended to perform better on the remaining components of reasoning about fractions.

Although past research has provided insights into teachers' understanding of RU, it has focused heavily on such understanding in fraction multiplication and division situations, given that RU change during the process (e.g., Izsák et al., 2019). Thus, these studies capture teachers' *flexibility with* RU, which can be defined as "a teacher's ability to keep track of the unit to which a fraction refers . . . and to shift their relative understanding . . . as the referent unit changes" (Lee, Brown, & Orrill, 2011, p. 204). Although fraction multiplication and division situations provide an invaluable opportunity to examine whether teachers can identify referent units correctly and think accordingly as the referent unit changes, we argue that RU are important in any fraction concept. Our argument is grounded in the view that understanding RU also includes *attention to* RU, even in less explicit situations. To illustrate what we mean by *attention to* RU, when comparing fractions, creating equivalent fractions, and performing fraction operations such as fraction addition and subtraction, the same referent unit is used for the fractions involved. For instance, when two fractions are added, both fractions refer to the same whole. Thus, *attention to* RU could capture another characteristic of teachers' understanding of RU.

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In sum, although prior work has provided evidence for the importance of RU in understanding fractions, we still know little about the relationship between different characteristics of RU. In particular, we hypothesized that in addition to flexibility with RU, attention to RU is an important characteristic of teachers' understanding of RU and, in general, of their overall performance on fractions. To test our hypothesis, we created two constructed-response problems, one capturing teachers' *attention to* RU in a fraction comparison situation and the other capturing teachers' *flexibility with* RU in a fraction multiplication situation involving a visual representation. By using data collected from 246 U.S. in-service teachers who were teaching mathematics in Grades 3–7, we examined the relationship between teachers' performance on these two problems and the extent to which teachers' professional background was related to their responses to these two problems. Finally, we explored how teachers' responses to these two problems were related to their overall performance on a fractions measure. We aimed to answer the following research questions:

1. To what extent do teachers pay attention to RU?
2. To what extent do teachers demonstrate flexibility with RU?
3. What is the relationship between teachers' attention to and flexibility with RU?
4. What aspects of teachers' professional background are related to their attention to and flexibility with RU?
5. To what extent are teachers' attention to and flexibility with RU, along with their professional background, associated with their overall performance on fractions?

Our study contributes to the current literature in three significant ways. First, prior work has not focused on the relationship between teachers' understanding of different characteristics of RU. Thus, by examining the relationship between teachers' attention to and flexibility with RU, we aimed to contribute teachers' understanding of RU and fraction operations. Second, limited research (Izsák et al., 2019) has investigated the relationship between teachers' professional background and their understanding of RU. Thus, knowing the extent to which teachers' professional background is associated with their attention to and flexibility with RU will have implications for mathematics teacher education. Finally, by investigating the relationship between teachers' understanding of RU and their performance on a fractions measure, we aimed to provide further evidence for how teachers' understanding of RU might be linked to their overall performance on fractions.

Theoretical Framework

Referent units can be defined as units number refer to in mathematical situations. Although it is possible for teachers and students to perform algorithms correctly without relying on RU, a conceptual understanding of fractions requires one to explicitly attend to the units and to be aware of the units in these situations (Philipp & Hawthorne, 2015). Let us illustrate the RU in two different problem situations:

1. Which fraction is larger: 1/3 or 1/2?
2. One serving of yogurt is 1/3 of a cup. For one meal, Amanda ate 1/2 of a serving. How many cups of yogurt did Amanda eat?

In the first problem, the answer can be found by finding a common denominator for both fractions and noticing that 2/6 is smaller than 3/6. However, the comparison makes only sense if both fractions refer to the same unit. Thus, attention to RU is necessary to develop a conceptual understanding in situations where the referent unit stays the same. In this way, teachers can overcome several misconceptions such as the larger the denominator, the larger the fraction or adding across numerators and denominators (Newton, 2008). In the second problem, however, the numbers refer to different units. Whereas 1/3 and the product, 1/6, refer to 1 cup, 1/2 refers to one serving, which is 1/3 of a cup. When performing the standard algorithm, the answer, 1/6, can be found by multiplying across numerators and denominators. On the other hand, a conceptual understanding of fractions requires showing flexibility with RU by understanding that the RU for 1/2 and 1/3 are different and thinking accordingly as the referent unit changes. Therefore, partitioning the serving size into two parts and shading one part is needed to show 1/2 of 1/3 (Figure 1b). Because the problem asks for the number of cups, the referent unit of 1/6 then becomes 1 cup, the whole rectangle (Figure 1c).

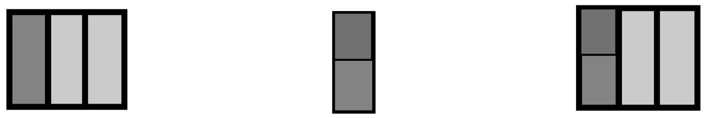


Figure 1: (a) 1/3 of the rectangle; (b) 1/2 of the 1/3; (c) 1/6 of the rectangle

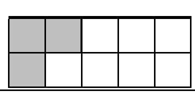
Most prior work on RU has focused on teachers' understanding of fraction multiplication and division, and reported both future and in-service teachers' struggle with RU (e.g., Baek et al., 2017; Izsák, 2008; Izsák et al., 2019; Lee, 2017; Webel et al., 2016). Much of this research used fraction multiplication and reported teachers' reliance on the overlapping method, which uses the same referent unit for the multiplier, multiplicand, and product. These studies have acknowledged that using the overlapping method either results in incorrect answers or causes mostly step-by-step algorithms instead of conceptual understanding about what it means to multiply two fractions.

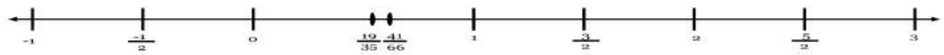
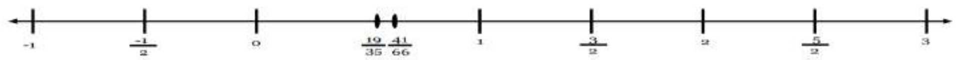
Methods

The data were collected from 246 in-service mathematics teachers in Grades 3–7 across 21 states in the United States. Teachers in our sample were mostly female (84%) and White (68.1%). In addition, 25.2% of the teachers had a master's degree, 77% of them were teaching mathematics in Grades 3–5, and 23% were teaching mathematics in Grades 6–7. While 70.3% had traditional certification, 19.3% had a credential in mathematics, and 52.5% were fully certified.

As seen in Table 1, the fractions measure used in this study consisted of a set of six items adapted from prior research (e.g., Siegler, 2015), the DTMR survey (Izsák et al., 2019) and the Teacher Education and Development Study in Mathematics (TEDS-M) survey (Tatto et al., 2012), and teacher education resources (Van de Walle, Karp, & Bay-Williams, 2019). We also administered the background survey (Izsák et al., 2019) and collected information regarding the professional background of our sample.

Table 1: Fractions measure items

Key concept	Item
Attention to RU	Is it possible for 1/3 to be greater than 1/2? Explain your thinking.
Equivalent fractions	In the figure, how many **MORE** small squares need to be shaded so that 4/5 of the total number of small squares are shaded? Explain your answer. 
Comparing fractions	For each set of fractions, put < , >, or = to make the statement true. $\frac{9}{21}$ $\frac{15}{21}$ $\frac{31}{57}$ $\frac{23}{57}$ $\frac{61}{44}$ $\frac{33}{44}$ $\frac{20}{17}$ $\frac{20}{33}$ $\frac{49}{48}$ $\frac{49}{47}$ $\frac{71}{60}$ $\frac{71}{52}$

Estimating the sum of fractions	The fractions $\frac{19}{35}$ and $\frac{41}{66}$ have been placed on a number line. **Without computing**, please estimate the sum of $\frac{19}{35}+\frac{41}{66}$ by placing a dot on the number line where you think the sum would be found. Explain your answer. 
Flexibility with RU	This item cannot be displayed because it is currently part of the DTMR survey (Izsák et al., 2019). We provided a drawn rectangle and asked teachers to model fraction multiplication and explain their answer.
Estimating the quotient of fractions	The fractions $\frac{19}{35}$ and $\frac{41}{66}$ have been placed on a number line. **Without computing**, please estimate the quotient of $\frac{41}{66}\div\frac{19}{35}$ by placing a dot on the number line where you think the quotient would be found. Explain your answer. 

We independently coded the items on attention to and flexibility with RU. The agreement was over 90% for each item. We classified teachers' responses to the item on attention to RU into three categories: no attention to RU, partial attention to RU, and full attention to RU. Specifically, teachers assigned to the first category did not refer to any RU implicitly or explicitly in their explanations. The second category included teachers who were using the same referent unit. The third category captured teachers who responded that the answer depended on the referent unit. We also classified teachers' responses to the item on flexibility with RU into three categories: no flexibility with RU, partial flexibility with RU, and flexibility with RU. The first category included teachers who did not demonstrate flexibility with RU at all in their responses such as "I am unsure how to model that the product of 1/3 × 1/4 is 1/12." The second category included teachers who used the overlapping method such as "She should draw two vertical lines to divide the rectangle into 3 equal-sized parts across, then shade in one of the vertical rectangles. The shaded piece that is overlapped demonstrates the 1/12." The third category included teachers who demonstrated flexibility with RU by keeping track of the units with explanations such as: "She should divide the picture into 3 equal-sized pieces vertically and show that 1/3 of the 1/4 is 1/12 of the whole." We also scored the remaining four fraction items and the agreement was greater than 90%.

To report teachers' attention to and flexibility with RU, we computed the percentages of responses in each category. To investigate the relationship between teachers' attention to and flexibility with RU, we used a Pearson chi-square test. We also computed the correlation between these categories by using gamma statistics, given that the categories for each problem were ordinal. To investigate the relationship between teachers' responses to the referent unit problems and their professional background variables, we ran a separate ordinal logistic regression for each problem. Finally, to examine the relationships among teachers' overall performance on other items of the fractions measure, their attention to and flexibility with RU, and the professional background variables, we ran a linear regression in which the total score was predicted by teachers' attention to and flexibility with RU and the aforementioned background variables.

Results

Teachers' Attention to Referent Units

As shown in Figure 2, 54.5% of the teachers demonstrated attention to RU by responding that 1/3 could be greater than 1/2, depending on the referent unit. For instance, one teacher explained that "If I am comparing two different-sized objects, then 1/3 may be greater than 1/2." On the other hand, 19.9% of the teachers demonstrated partial attention to RU by reporting that 1/3 could not be greater than 1/2 and by explicitly using the same referent unit to justify their responses. Furthermore, 25.6% of the teachers did not demonstrate attention to RU (Figure 2). Specifically, 57% of these teachers did not provide any explanation that showed why 1/3 could not be greater than 1/2, whereas 25.4% of the teachers constructed equivalent fractions in their explanations. For example, one teacher wrote "To easily compare these fractions, you can find common denominators, 2/6 and 3/6. The one half will always be greater than the one third." Lastly, 17.6% of the teachers either made factual statements in their explanations without mentioning any referent unit or they converted fractions into percentages by reporting that 1/3 and 1/2 means 33% and 50%, respectively.

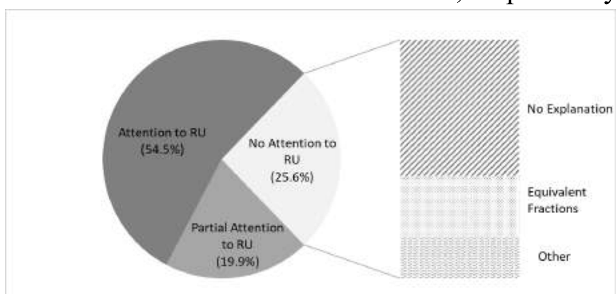


Figure 2: Teachers' performance on the item that measured their attention to RU

Teachers' Flexibility with Referent Units

Teachers' responses to the flexibility with RU item suggested that only 11.8% of the teachers demonstrated flexibility with RU (Figure 3). Those teachers reported that the referent unit for 1/4 was the entire rectangle and that the referent unit for 1/3 was 1/4 of the rectangle (i.e., the shaded part), not the entire rectangle. They also pointed out that 1/12 was 1/3 of the 1/4 rectangle. For example, one teacher explained "divide the picture [1/4 of the given rectangle] into 3 equal-sized pieces vertically and show that 1/3 of the 1/4 is 1/12 of the whole." On the other hand, the remaining teachers (88.2%) appeared to struggle demonstrating flexibility with RU. In particular, 44.3% of the teachers demonstrated partial flexibility with RU by relying on the overlapping method. They did not specify different RU for 1/3 and 1/4, and their explanations implied that for both 1/3 and 1/4, they considered the entire rectangle as their referent unit. For instance, one teacher explained that "Divide the rectangle vertically into 3 equal-sized parts and shade in one part. The overlapping part between the horizontally shaded part and vertically shaded part (one square) is 1/12." Unlike the aforementioned two categories, 43.9% of the teachers did not demonstrate any flexibility with RU. Those teachers did not appear to consider any referent unit, and they did not provide explanations for each fraction.

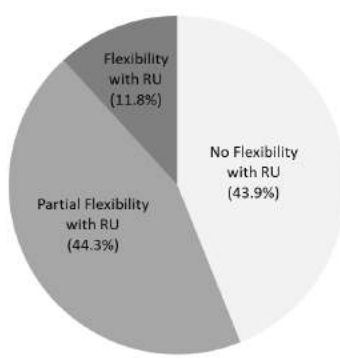


Figure 3: Teachers' performance on the item measuring their flexibility with RU

Relationship between Attention to and Flexibility with Referent Units

We found a significant, but moderate relationship between relationship between teachers' attention to and flexibility with RU ($\chi^2(4) = 13.3$, $p = .01$; $G = .35$). As shown in Figure 4, 60.3% of the teachers who did not pay attention to RU failed to demonstrate flexibility with RU, whereas 35.1% of the teachers who paid attention to RU failed to demonstrate flexibility with RU.

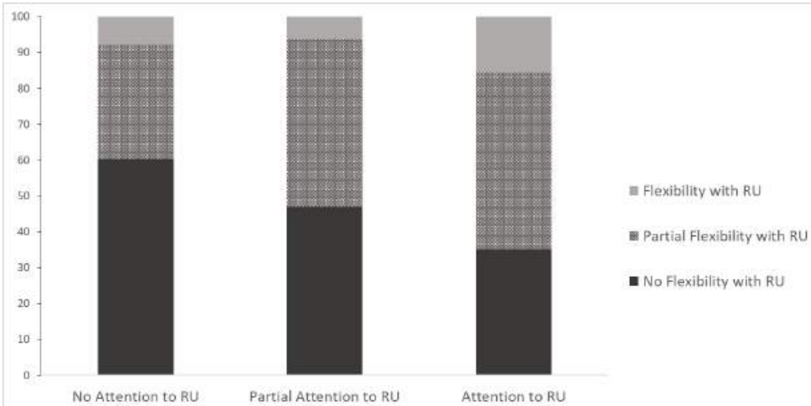


Figure 4: Teachers' performance on for different levels of attention to RU

Relationship Between Understanding of Referent Units and Professional Background

We also examined the relationships between teachers' understanding of RU and their various professional background variables. As shown in Table 2, none of the variables for teachers' background was associated with their attention to RU, whereas middle grades teachers and traditionally certified teachers showed more flexibility with RU compared with upper elementary and non-traditionally certified teachers. For example, the odds of middle grades teachers showing flexibility with RU was 2.67 times higher than that of elementary grades teachers ($p = .001$). This means that middle grades teachers were 2.67 times more likely to demonstrate flexibility with RU than elementary grades teachers.

Table 2: Logistic Regression of Probability of Attention to and Flexibility with RU

Teachers' professional background	Attention to RU	Flexibility with RU
Number of mathematics content courses (3 or more)	0.880 (.23)	0.667 (.18)
Number of mathematics methods courses (3 or more)	1.084 (0.32)	1.088 (0.32)
Fully certified teachers	0.875 (0.24)	0.923 (0.25)
Middle school mathematics teachers (Grades 6 & 7)	0.983 (.30)	2.666** (.81)
Traditionally certified teachers	1.265 (0.38)	2.098* (.64)

Note. Odds rations shown. Standard errors are in parentheses. *$p < 0.05$, **$p < 0.01$.

Relationship Between Knowledge of Referent Units and Fractions

As shown in Table 3, teachers' attention to and flexibility with RU significantly predicted their overall performance on the fractions measure. Specifically, when teachers' attention to and flexibility with RU were entered into the model separately, teachers who paid attention to RU significantly outperformed those who did not pay attention (effect sizes of .41 and .58 for the partial attention to and attention to referent unit categories, $p = .035$ and $p < .0001$). Similarly, those who demonstrated partial flexibility or flexibility with RU also performed significantly better on the fractions measure compared with those who did not show flexibility with RU (effect sizes of .52 and .60 for teachers who were in the groups showing partial flexibility and flexibility with RU, $p < .0001$ and $p = .007$).

Table 3: Teachers' performance on fractions predicted by their Attention to and Flexibility with Referent Units and Professional Background

	Attention to RU	Flexibility with RU	Attention to and flexibility with RU, and professional background
Attention to and flexibility with RU			
Partial attention to RU	0.140* (.067)		0.142* (.064)
Attention to RU	0.198*** (.053)		0.175*** (.052)
Partial flexibility with RU		0.171*** (.047)	0.105* (.048)
Flexibility with RU		0.198** (.073)	0.115 (.072)
Professional background			
Number of mathematics content courses			−0.042 (.046)
Number of mathematics methods courses			0.059 (.50)
Fully certified teachers			−0.024 (.046)
Middle school mathematics teachers			0.206*** (.053)
Traditionally certified teachers			0.039 (.052)

Note. $N = 238$ for all models. The numbers in parentheses are standards errors. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

Finally, when teachers' attention to and flexibility with RU were included in the model along with their professional background variables, teachers who demonstrated partial attention to RU or those who demonstrated full attention to RU still performed better than those who did not demonstrate attention to RU (effect size of .43 and $p = .028$ for the partial attention to RU category; and effect size of .53 and $p = .001$ for the full attention to RU category). However, teachers' flexibility with RU did not seem to be significantly correlated with their overall performance on the fractions measure. This is possibly because of the correlation we reported earlier between teachers' professional background variables and their flexibility with RU. Of these variables, the only significant predictor of teachers' overall performance was being a middle grades teacher. Indeed, the difference between elementary and middle grades teachers' performance was an effect size of .63, $p < .001$. Other variables, such as the number of courses or being fully certified, did not link to their overall performance on fractions.

Discussion

In the present study, we examined U.S. in-service teachers' attention to and flexibility with RU and the relationship between these two characteristics, along with how teachers' understanding of RU was linked to their professional background and performance on the fraction items. We found that although about half of the teachers paid attention to RU, only 12% of the teachers showed flexibility with RU, which suggests that showing flexibility with RU is a more difficult concept to grasp. Our findings regarding teachers' flexibility with RU are similar to those from prior work (e.g., Lee et al., 2011; Webel et al., 2016). Furthermore, in alignment with past research (e.g., Izsák, 2008; Lee et al., 2011; Webel et al., 2016), teachers in our study commonly used the overlapping method to model fraction multiplication, indicating these teachers' difficulty with making sense of fraction multiplication.

Furthermore, our findings suggest a significant, but moderate relationship between teachers' attention to and flexibility with RU. These results may provide initial evidence that these items capture different characteristics of teachers' understanding of RU. It is interesting that teachers' performance on the item measuring flexibility with RU was associated with the teachers' preparation route, whereas the item measures attention to RU was not associated with any teacher background indicators. This may be because teacher education programs focus more on modeling fraction multiplication and division, given that many studies on future teachers have focused on fraction multiplication (e.g., Baek et al., 2017).

In a similar vein, it is important to point out that the number of mathematics content and methods courses was not associated with teachers' attention to and flexibility with RU. In an extensive review, Olanoff et al. (2014) reported an urgent need for research that finds ways to improve future teachers' understanding of fractions. The present study suggests that emphasizing attention to RU in teacher preparation programs, even when the referent unit stays the same, could help future teachers improve their understanding of fractions.

Our findings also underscore the importance of teachers' attention to and flexibility with RU in relation to their performance on other fraction concepts. In particular, teachers who paid attention to RU performed better than those who did not. Similarly, teachers who demonstrated flexibility with RU performed better on other fraction concepts than those who did not demonstrate such flexibility. Furthermore, when both attention to and flexibility with RU were included together, in addition to teachers' professional background variables, teachers who paid attention to RU or those who used the overlapping method for fraction multiplication performed better on the remaining items of the fractions measure than did those who did not pay attention to RU or those who showed no flexibility with RU. However, teachers who showed full flexibility with RU did not perform well compared with those who did not show any flexibility after adjusting for attention to RU. In sum, these findings also confirm the importance of teachers' understanding of RU in their mastery of other fraction concepts.

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