CLASSROOM EVENTS ON PROBLEM SOLVING WITH GEOGEBRA ANTICIPATED BY FUTURE MATHEMATICS TEACHERS

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This paper presents an analysis of the classroom events that a group of future teachers of Secondary Education mathematics identified from their own experience when using GeoGebra to solve problems. The data analyzed are from the written materials by twelve mathematics majors who were studying the Mathematics for Teaching course. The students, in pairs, solved three problems using GeoGebra. They were then asked to posit three events that could arise if their students were to use GeoGebra to solve problems. After analyzing the events presented, they were classified in terms of mathematical reasoning, mathematical creation and techno-mathematical ability.

Key words: Teacher training, problem solving, technology, classroom situations.

Introduction

Teacher training is a complex field of study that ranges from identifying the knowledge required to teach a discipline, to proposing strategies for developing that knowledge. The incorporation of technology into the process of teaching and learning mathematics poses new challenges in teacher training, particularly when defining training programs. What kinds of activities should be conducted during the training period of a mathematics teacher? What technological tools should be used? How should they be used? How does the use of technology influence their initial training?

On the one hand, the appearance of a certain type of technology has expanded the set of tools available to teachers to respond to events or contingencies that occur in the classroom. For example, Rowland and Zazkis (2013) analyze possible actions in response to a hypothetical answer from a student who is asked to provide a fraction between ½ and ¾. The student answers 2/3, stating that for the numerator she chose 2 because it is between 1 and 3, and for the denominator she chose 3 because it is between 2 and 4. One of the options, presented by the authors, for incorporating the student's idea into the classroom proposes representing, with the aid of technology, a geometric situation that can intuitively provide an answer. This would be done by representing the fractions as the slopes of lines that pass through the origin and through a coordinate point (n, d), where n is the numerator and d the denominator of each fraction.

On the other hand, the use of technological tools in the classroom gives rise to certain types of situations that would not appear in another context. Wasserman, Zazkis, Baldinger, Marmur, & Murray (2019) provide an example in which a group of students used a MAPLE command to confirm that a number is prime. When an operation was entered as the input argument (14:2), MAPLE indicated that it was not a prime. This would not have happened if the input had been a number (7). The authors note that this contingency could be exploited to interact with students and discuss the importance of differentiating number sets and conceptualizing multiplication with rational numbers. Hernández, Perdomo-Díaz and Camacho-Machín (2018) present an analogous situation, in this case by using GeoGebra. This program does not provide an answer when the Tangents tool is used, entering as input values two points, one outside the circle and the other on the circumference. This situation offers the opportunity to discuss, with students, questions such as: Why is nothing happening? What important properties related to lines tangent to a circle do these values not consider? That is, the contingency could be leveraged to have a discussion with students on the properties of lines tangent to a circle and how GeoGebra processes them when plotting these lines.

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GeoGebra is one of the most widely used technological tools for teaching and learning mathematics. Its greatest potential lies in the fact that it is a dynamic geometry software (DGS) package that is dynamic enough to analyze questions, conjectures, discover mathematical properties and establish connections between known properties (Jacinto & Carreira, 2017; Sánchez-Muñoz, 2011; Santos-Trigo & Camacho-Machín, 2013). Given this context, teachers must be familiar with the program and have experience using it so as to take advantage of the opportunities that technology offers to gain mathematical knowledge (Camacho-Machín & Santos-Trigo, 2016). Teachers must also possess a certain ability and expertise to deal with the mathematical questions, doubts, and interpretations that arise every day in classrooms (Conner, Wilson, & Kim, 2011). When an event occurs in the classroom, the teacher must decide whether to ignore it, set the issue aside after considering it or try to incorporate it into the class, which in many cases has a certain improvisational component (Rowland & Zazkis, 2013). For Conner, Wilson and Blume (2011), making a good decision requires a certain type of knowledge and skill. It takes "a particular kind of expertise which includes a deep mathematical knowledge that allows them to recognize the opportunity, weigh its merits, and skillfully pursue or dismiss the opportunity" (p. 979).

An analysis of the problem-solving process with GeoGebra that an individual employ can be used to identify interesting situations to propose as activities for training teachers (Camacho-Machín, Perdomo-Díaz, & Hernández, 2019). But it is also interesting to see what situations future teachers imagine could occur when they ask their students to use GeoGebra to solve problems. Anticipating possible events or contingencies that could come up in class is one way to develop teaching skills (Carrillo, 2015). Consequently, analyzing contingencies and anticipating exercises can provide a connection between the training of future mathematics instructors and teaching in Secondary Education. This leads to the question that prompted this research: What kind of classroom situations do future secondary education mathematics teachers anticipate from their own experience using GeoGebra to solve problems?

The work presented here consists of an exploratory study that addresses this question. The research was conducted with a group of mathematics majors who were taking the "Mathematics for Teaching" course. One part of this course consisted of analyzing classroom situations proposed by Heid, Wilson, & Blume (2015), and another part of the course involved doing a Problem-Solving Workshop using GeoGebra. As part of the final activity in this workshop, the students were asked to identify situations that might arise in a secondary education math class in which GeoGebra is utilized to solve problems. The goal of this research is to analyze classroom situations involving the use of GeoGebra to solve problems anticipated by the participants.

Conceptual Framework

The reference for this research is a framework developed from real and hypothetical situations involving secondary education mathematics classes known as Mathematical Understanding for Secondary Teaching (MUST) (Heid, Wilson, & Blume, 2015). This framework considers that the mathematical understanding that an individual need to teach the discipline in high school can be described from three different, closely interrelated, perspectives: mathematical proficiency, mathematical activity and mathematical context. The first perspective focuses on "knowing" mathematics, the second on "being able to do" mathematics, and the third on the ability to "adjust" that knowledge and know-how to secondary education students (Kilpatrick et al., 2015). As these authors point out, this particular understanding of mathematics, typical of a teacher's endeavor, has a dynamic character, since it starts to take shape based on the understanding that a teacher would have as a student, before developing and transforming during the teacher's training and subsequent career.

When analyzing mathematical understanding for teaching from the perspective of mathematical proficiency, the focus is on mastering the content being taught and the ability to make connections between the concepts to be taught and other mathematical content. This perspective thus includes components such as conceptual understanding, proficiency with procedures, strategic competence and flexible reasoning (Kilpatrick, 2015). For Camacho-Machín and Santos-Trigo (2015), the development of these four components is essential for learning that is characterized by constant questioning and problem solving.

From the perspective of mathematical activity, mathematical understanding for teaching comprises the set of specific mathematical actions that an instructor performs while teaching. Zbiek and Heid (2018) advocate for a teaching practice in which the mathematical activity is made explicit, such that the discipline can be extended beyond the content of the subject, consisting of procedures and concepts. The MUST model defines this perspective from three interrelated components (Kilpatrick et al., 2015):

- Mathematical perception: Groups the actions of recognizing and identifying the mathematical characteristics specific to the different structures, the different notations or symbolic forms, as well as the ability to ascertain when a mathematical argument, whether expressed simply or rigorously, is valid, and the ability to connect mathematical ideas with each other (representing ideas in different structures and connecting various concepts) and with the real world (explaining physical problems through mathematics).
- Mathematical reasoning: Groups the observe, conjecture and justify or prove activities by using deductive logic, mathematical properties, regularities and patterns, generalizations of specific cases, restricting properties and extensions to other structures.
- Mathematical creation: Implies the ability to find new paths to express mathematical objects, generate new ones and transform their representation. This is related to choosing representations of objects that highlight their structure, restrictions or properties, when new objects are defined and when they are manipulated by changing their form, but not their representation.

Finally, the mathematical context perspective includes aspects of mathematical understanding that come into play exclusively in the teaching profession, such as recognizing the mathematical nature of students' questions and errors, or recognizing when an argument or solution provided by a student is incomplete or satisfies the conditions of a problem (Kilpatrick, et al., 2015). Among its components, the authors include: synthesizing mathematical ideas, interacting and understanding students' mathematical thinking, knowing and using the curriculum, evaluating students' mathematical knowledge, and reflecting on the mathematics employed in the classroom.

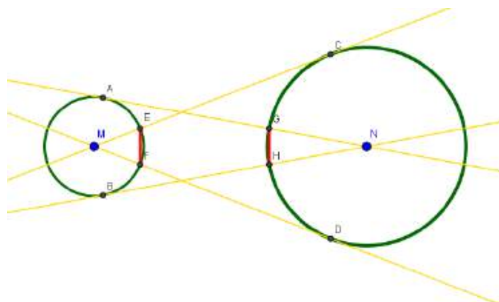
This research combines the mathematical context and mathematical activity perspectives. The former was taken into account in the design of the tasks proposed for the students; specifically, the last component was emphasized by reflecting on the mathematics employed in the classroom, asking students to anticipate *Situations* that could arise in a Secondary Education class where GeoGebra is used to solve problems. For MUST framework, a *Situation* is "a way of capturing classroom practice […] portrays an incident that occurred in the context of teaching secondary mathematics in which some mathematical point is at issue" (p. 4). In each *Situation* a *Prompt* and a set of *Mathematical Foci* can be distinguished. A *Prompt* is something that has occurred or may occur in the context of teaching mathematics, such as a student' question, or a mathematical fact that a student has identify. The *Prompts* proposed by participants were analyzed from the mathematical activity perspective, using their three components as the basis for classifying them.

Methodology

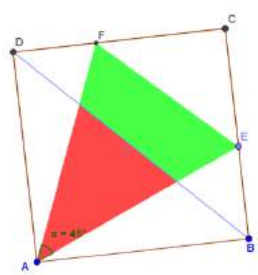
The participants in this research were the students in the "Mathematics for Teaching" course, which is offered as an elective for senior-year mathematics majors at the University of La Laguna (Spain). The main goal of this course is to develop the students' theoretical, practical and instrumental skills associated with the activity of teaching mathematics at the high school and university levels. This includes knowing and using heuristic strategies for solving math problems, as well as technological tools for teaching and learning mathematics.

The data were collected in the 2017-2018 school year over the course of four weeks, in which the students devoted half the class time to analyzing classroom situations designed under the MUST framework (Heid, Wilson & Blume, 2015), and the other half to participating in a GeoGebra Problem Solving Workshop. The workshop consisted of eight, two-hour sessions. In the first five, the students solved the following three problems:

Equal chords. Given two circles with centers M and N, lines are drawn from the center of each that are tangent to the other. The points where the tangents lines intersect the circumferences define two chords, EF and GH. Prove that the length of the chords is the same.



45º angle. Given a square ABCD, draw a 45º angle inside the square, with its vertex at A. This yields two rays that cut the sides opposite A at points E and F (see drawing). Study the relationship between the two parts when triangle AEF is divided by the diagonal BD.



Connect Islands. We want to connect three islands (A, B, and C) with a fiber optic network in a way that uses the least amount of cable. The distances between the islands are 79,322 m (A-B), 64,514 m (A-C) and 95,932 m (B-C). Where should the connection point be located to minimize the amount of cable needed?

In the last three sessions, students focused on identifying and analyzing *Situations*, in the sense of MUST framework, that could happen in a high school mathematics class in which GeoGebra is used to solve the above problems. The data analyzed in this paper are from the report that the students had to submit as a product of those last sessions. The instructions given to the students were:

- Write *Situations* resulting from the use of technology in solving workshop problems.
- In each *Situation* indicate at least three *Prompts*, using GeoGebra to show how they arise.
- Select a *Prompt* for each problem and identify a set of mathematical foci relevant to the prompt.

This activity was intended for students to establish connections between the problem solving activities with GeoGebra that they had carried out in the workshop and the analysis of classroom situations that they had done in the parallel sessions. In this way, the participants moved from a role as students to a role as teachers, taking their own experience using GeoGebra to solve problems as a reference point to reflect on possible situations that may occur in a mathematics classroom where this type of didactic resources.

The course was taken by 18 students, who worked in pairs. For this paper, only the reports from six of the student pairs were selected. The selection criterion was that the students must have handed in all the tasks from the workshop and the study correctly and on time.

The analysis process consisted of identifying what role GeoGebra played in each of the *Prompts* indicated by the participants, and what components of mathematical understanding for teaching that MUST proposes from the perspectives of mathematical activity underlies those prompts. From there, types of class situations were defined that future teachers anticipate from their own experience using GeoGebra to solve problems.

Data analysis

An analysis of the content of the reports prepared by the six students pairs allowed us to identify three types of *Prompts*:

Type 1: The focus is on giving a mathematical explanation to the operation of a GeoGebra tool. These *Prompts* have to do with the development of techno-mathematical ability (Jacinto and Carreira, 2017).

Type 2: *Prompts* that involve justifying or demonstrating a mathematical property that has been observed when making a dynamic construction with GeoGebra.

Type 3: *Prompts* where a conjecture is formulated and GeoGebra is used to build new elements that allow it to be proven or rejected.

The last two types of *Prompts* are closely related to mathematical reasoning and mathematical creation, which are components of mathematical understanding for teaching from the perspective of mathematical activity (Kilpatrick, et al., 2015).

Type 1: The *Prompts* in this category arise directly from the use of tools implemented in GeoGebra and from an understanding of the mathematics on which it is based. Four of the student pairs identified *Prompts* of this type (Table 1). Three of them (P1, P3 and P5) proposed an event related to the use of the Tangents tool that yields an answer they understand; however, it could lead to a classroom situation where a student asks about the steps GeoGebra performed to give that answer. Pair P8 identified an event in which students would question an answer provided by the DGS.

Table 1: Prompts related to the mathematical meaning of actions in GeoGebra

Problem	Topic	Summary of the Prompt	P1	P2	P3	P5	P7	P8
Equal chords	Tangent lines	Can a tangent line to a circle be drawn from an outside point without using the *Tangents* tool?	■	-	■	■	-	-
Equal chords	Tangent lines	Using the *Line* tool, a line tangent to a circle is constructed from an outside point. Why does it seem to satisfy the tangency property when it actually does not?	-	-	-	-	-	■

Type 2: This category includes *Prompts* that relate to the need to prove or demonstrate mathematical conjectures arising from the use of GeoGebra. These hypothetical events related to the action of seeking a justification, whether formal or not, describe a classroom situation in which a secondary education student discovers a property while solving a problem with GeoGebra and asks

the teacher about its veracity or gives an argument that requires formalization. This is the category into which most of the *Prompts* were classified, a total of 10 (Table 2), although all were proposed by three of the student pairs (P1, P3 and P5).

Table 2: Conjectures from Prompts related with mathematical reasoning

Problem	Topic	Summary of the conjecture	P1	P2	P3	P5	P7	P8
Equal chords	Tangent lines	A line tangent to a circle forms an angle of 90° with the radius that connects the center to the point of tangency.	-	-	■	-	-	-
Equal chords	Bisector	The two triangles that are formed by a chord, the two radii it connects and the bisector of these radii are congruent.	■	-	-	■	-	-
Equal chords	Bisector	The bisector of the two lines tangent to a circle from an outside point passes through the center of the circle.	■	-	■	-	-	-
Equal chords	Similarity	Two overlapping triangles (with a common angle) are similar, although they are not in the Thales position.	-	-	-	■	-	-
45° angle	Invariant	Given a family of triangles AEF, where angle A measures 45°, and inscribed in a square ABCD with side l, such that A is on a vertex and E is on side BC, then the height from A of any triangle in the family is constant and measures l.	-	-	■	■	-	-
45° angle	Diagonal of the square	Given a point P on the diagonal that joins two vertices of a square, draw two segments to the other two vertices. This yields four triangles. The heights of the triangles from P match pairwise.	■	-	-	■	-	-
Connect islands	The Fermat point	The Fermat point is the point of intersection of the three circles in which the equilateral triangles used to build it are inscribed.	-	-	-	■	-	-
Connect islands	The Fermat point	The Fermat point, along with three vertices of one of the equilateral triangles used to construct it, form a cyclic quadrilateral.	-	-	■	■	-	-
Connect islands	The Fermat point	The three segments that connect the Fermat point of a triangle with the vertices of the equilateral triangles used to construct it form angles of 120° with one another.	-	-	■	-	-	-
Connect islands	Rotations	Given two points A and B, the triangle formed by A, B and the point resulting from rotating B 60° with respect to A is equilateral.	-	-	■	-	-	-

Type 3: This last category includes the *Prompts* submitted by the student pairs who propose using GeoGebra as a resource to check a previous mathematical idea. Eight *Prompts* of this type were identified (Table 3), presented by four of the pairs (P1, P2, P7 and P8). Unlike the previous category, the formula to describe the *Prompt* starts from a situation in which the high school students have an idea, and then resort to technology to test it. This approach induces a change in the follow-up actions. An existing mathematical idea has to be transferred to a dynamic construction such that the mathematical property to be verified is represented and emphasized. This type of proposal would provide a starting point to develop mathematical creation.

Table 3: Properties from Prompts related to mathematical creation

Problem	Topic	Summary of the mathematical property	P1	P2	P3	P5	P7	P8

Equal chords	Tangent lines	Given two circles and the lines tangent to them from the centers of the other, the chords formed by the tangency points are equal, as are the lines formed by the points of intersection with the secant lines.	-	■	-	-	-	-
Equal chords	Bisector	The bisector of an angle divides any triangle limited by the rays into two congruent triangles.	-	-	-	-	■	-
45° angle	Invariant	The area of a family of polygons inscribed in a triangle is constant.	■	-	-	-	-	-
45° angle	Diagonal of the square	Given two similar triangles, there is a proportionality ratio between their areas.	-	■	-	-	-	-
45° angle	Invariant	Given a rectangle, draw a 45° angle at one of its vertices so as to make a triangle with the points of intersection on the non-contiguous sides. The diagonal of the rectangle divides the triangle into two surfaces with the same area.	-	-	-	-	■	■
Connect islands	The Fermat point	The Fermat point minimizes the sum of the distances to the vertices for any triangle.	■	-	-	-	■	-
Connect islands	The Fermat point	The three segments that join the Fermat point of a triangle with the vertices of the equilateral triangles used to construct it have the same length.	-	■	-	-	-	-
Connect islands	Significant points	In any triangle, the sum of the distances from the vertices to the circumcenter is the minimum possible. (Idem with barycenter, orthocenter, incenter)	-	-	-	-	-	■

This type of *Prompt* includes minor questions that extend or stray from the solution to the original problem. In general, delving into these questions could be useful to segue or connect to other mathematical results. A relevant exercise to prepare for future contingencies is knowing the various branches that originate from a problem and reflecting on potential connections to mathematics. From the point of view of Mathematical Context, the participants showed an ability to reflect on the mathematics of teaching practices, a necessary skill in the classroom (Heid, Wilson, & Blume, 2015).

Final discussion

The main objective of the reflection task proposed to the students in the "Mathematics for Teaching" course was to place them in the teacher's role after having solved a set of problems using GeoGebra. This activity was proposed as a way to anticipate potential contingencies that they would have to deal with in a classroom where GeoGebra is used to solve problems. The goal was to develop their mathematical understanding for teaching from the perspective of the Mathematical Context of Teaching, a component of which is reflecting on the mathematics of teaching practices (Heid, Wilson, & Blume, 2015).

All the student pairs proposed at least one event involving the use of technology for each of the problems. This shows that the participants, in this initial stage of their training as teachers, developed to a certain degree their ability to reflect on the mathematics of teaching practices (Kilpatrick, et al., 2015). However, there are some differences between the pairs. Just P1 has indicated *Prompts* of all three types; two couples indicated just type 1 and type 2 *Prompts* (P3, and P5); one pair gave types 1 and 3 *Prompts* (P8); the other two students pairs' report just include type 3 *Prompts* (P2, and P7). Since all three types of events are situations that can occur in a math class using Geogebra to solve problems, it would be desirable for future teachers to be able to identify, anticipate, and analyze them. The previous results show that this is not something that always happens, which points to the need for training that offers opportunities to reflect and deepen this type of analysis.

The group of proposals categorized as Type 1 included *Prompts* that have to do with classroom contingencies that could be used to interact with students and discuss mathematical meanings of technological facts (Wasserman, et al., 2019). The didactic steps to answer the questions that are proposed as a starting point of the *Prompts* would be the same in a university course as in a Secondary Education classroom.

As concerns the other two types of *Prompts* in our categorization (Types 2 and 3), we observed that the proposals included here were the most numerous. This could be related to the types of situations that each pair encountered while they were solving the problems themselves, during the first three tasks of the GeoGebra Problem-Solving Workshop, which would underscore the close relationship that exists between the different perspectives of Mathematical Understanding (Kilpatrick, et al., 2015).

The inclusion of technology as a tool for mathematical work in the classroom entails a change in the types of situations that a teacher must face in the classroom. This must be accompanied by a change in the types of training activities offered to future teachers of mathematics. The analysis conducted as part of this research shows how tasks involving reflection on one's own experience contribute to the development of mathematical understanding for teaching the discipline, particularly in Secondary Education.

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References

Camacho-Machín, M., Perdomo-Díaz, J., & Hernández, A. (2019). Actividades para la formación de profesores derivadas del uso de GeoGebra en la resolución de problemas. In E. Badillo, N. Climent, C. Fernández, & M. T. González (Eds.), *Investigación sobre el profesor de matemáticas: práctica de aula, conocimiento, competencia y desarrollo profesional.* (pp. 371-394). Salamanca: Ediciones Universidad de Salamanca.

Camacho-Machín, M., & Santos-Trigo, M. (2015). Aportes sobre resolución de problemas, tecnología y formación de profesores de matemáticas. In N. Planas (Ed.), *Avances y Realidades de la Educación matemática*, (pp. 113-131). Barcelona: GRAÓ.

Carrillo, J. (2015). Estudio personal compartido sobre conocimiento y desarrollo profesional del profesorado y resolución de problemas. In N. Planas, *Avances y Realidades de la Educación Matemática* (pp. 133-151). Barcelona: GRAO.

Conner, A., Wilson, P., & Kim, H. (2011). Building on mathematical events in the classroom. *ZDM Mathematics Education, 43*, 979-992.

Heid, M., Wilson, P. S., & Blume, G. W. (Eds.). (2015). *Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations.* United States of America: NCTM and IAP.

Hernández, A., Perdomo-Díaz, J., & Camacho-Machín, M. (2018). Prompts, Technology and Problem Solving. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education, 5*, p. 61. Umeå.

Hernández, A., Perdomo-Díaz, J., & Camacho-Machín, M. (2018). Propuesta de Uso de Tecnología en la Resolución de Problemas para la Formación Docente. Investigación en *Educación Matemática XXII* (pg. 633). Gijón: Ediciones de la Universidad de Oviedo. Taken from http://www.seiem.es/pub/actas/index.shtml

Jacinto, H., & Carreira, S. (2017). Mathematical Problem Solving with Technology: The Techno-Mathematical Fluency of a Student-with-GeoGebra. *International Journal of Science and Mathematics Education*, 1115-1136. doi:10.1007/s10763-016-9728-8

Kilpatrick, J. (2015). Background for the Mathematical Understanding Framework. In M. K. Heid, P. Wilson, & G. W. Blume (Eds.), *Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations* (pp. 1-8). Charlotte: Information Age Publishing Icc. and NCTM.

Kilpatrick, J., Blume, G., Heid, M. K., Wilson, J., Wilson, P., & Zbiek, R. M. (2015). Mathematical Understanding for Secondary Teaching: A Framework. In M. K. Heid, P. Wilson, & G. W. Blume (Eds.), *Mathematical*

Understanding for Secondary Teaching: A Framework and Classroom-Based Situations (pp. 9-30). Charlotte: Information Age Publishing Icc. and NCTM.

Rowland, T., & Zazkis, R. (2013). Contingency in the Mathematical Classroom: Opportunities Taken and Opportunities Missed. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 137-153. doi:10.1080/14926156.2013.784825

Sánchez-Muñoz, J. M. (2011). Visualización de Lugares Geométricos mediante el uso de Software de Geometría Dinámica GeoGebra. *Pensamiento Matemático*.

Santos-Trigo, M., & Camacho-Machín, M. (2013). Framing the use of computational technology in problem solving approaches. *The Mathematical Enthusiast, 10* (1&2), 279-302.

Santos-Trigo, M., & Camacho-Machín, M. (2016). Digital Technologies and Mathematical Problem Solving: Redesigning Resources, Materials, and Extending Learning Envioronments. In K. Newton (Ed.), *Problem-Solving: Strategies, Challenges and Outcomes* (pp. 31-50). Hauppauge, New York: Nova Science Publishers, lnc.

Wasserman, N. H., Zazkis, R., Baldinger, E., Marmur, O., & Murray, E. (2019). Points of Connection to Secondary Teaching in Undergraduate Mathematics Courses. *22nd Annual Conference on Research in Undergraduate Mathematics Education.* Oklahoma City.

Zbiek, R. M., & Heid, M. (2018). Making Connections from the Secondary Classroom to the Abstract Algebra Course: A Mathematical Activity Approach. In N. H. Wasserman, *Connecting Abstract Algebra to Secondary Mathematics, for Secondary Mathematics Teachers* (pp. 189-2019). Cham: Springer Nature Switzerland AG. doi:10.1007/978-3-319-99214-3_10