ELEMENTARY STUDENTS AND THEIR SELF-IDENTIFIED EMOTIONS AS THEY ENGAGED IN MATHEMATICAL PROBLEM SOLVING

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In this study, we investigated two students', ages ten and eleven, emotions while they engaged in mathematical problem solving. During three task-based interviews, the students explored parts of the unsolved problem the Graceful Tree Conjecture. While they were engaged in the interviews, they self-identified the emotions of frustration and joy they were feeling using the Wong-Baker Scale. The students displayed the interplay of the emotions of frustration and joy or which we consider to be productive struggle. A descripted case of Georgia is included to describe her emotions while problem solving.

Keywords: Affect, Emotion, Problem Solving

Past researchers have documented that students experience both positive and negative emotions while engaged in mathematics (Hannula, 2015; O'Dell, 2017) and Else-Quest, Hyde, and Hejmadi (2008) and Williams (2002) have found an association between students having positive emotions during problem-solving and the development of mathematical understanding. However, much of the research completed on emotions have been documented through surveys and not while students are engaged in mathematical problem solving (Hannula, 2015).

O'Dell (2017) documented that when students are experiencing the emotion of frustration followed by the emotion of joy while the student is engaged in mathematical problem solving, they are experiencing productive struggle. Struggle is when "students expend effort to make sense of mathematics, to figure something out that is not immediately apparent" (Heibert & Grouws, 2007, p. 387). It has been acknowledged allowing students the opportunity to struggle is beneficial (Hiebert & Grouws, 2007; Kapur, 2010). Kapur (2010) said when students are allowed to struggle they are able to significantly outperform students of a similar ability who have not been granted the opportunity to struggle. Kapur further found when a student has engaged in productive struggle they are able to better transfer that knowledge to mathematical concepts to which they have not yet been exposed.

Warshauer (2015) and Zeybek (2016) stated researchers know productive struggle is beneficial but there is limited research on what productive struggle looks like. O'Dell (2017) has documented that productive struggle is the interplay of the emotions of frustration and joy, but we want to examine first how students express the emotions of joy and frustration, second how they self-identify their emotions of frustration and joy, and lastly if they self-identify as having more frustration during problem solving, do they then experience more joy.

With these ideas, the following questions guided our research study:

1. How do students display the emotions of joy and frustration while they are engaged in problem solving?
2. How do students self-identify the emotions of frustration and joy they experience while problem solving?

Theoretical Framework

To examine how students display and identify emotions, we draw on the concept of positioning theory (Van Langenhove & Harré, 1999). Positioning theory is "the study of local moral orders as ever-shifting patterns of mutual and contestable rights and obligations of speaking and acting" (van

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Langenhove & Harré, 1999, p. 1). Thus, positioning theory allowed us to examine how students position themselves through their engagement with the mathematics and the other participants. In mathematics education positioning theory has been used to examine social interactions (e.g., Turner, Dominquez, Maldonado, & Empson, 2013; Wood, 2013; Yamakawa, Forman, & Ansell, 2009) and researchers have identified that students' emotions are linked to their positioning (e.g. Daher, 2015; Evans, Morgan, & Tsataroni, 2006). Further, Wood (2013) used positioning theory to examine micro-level moment-to-moment interactions that allow a researcher to document the exact moment an identity was enacted. A person positions themselves through conversations, actions, and dispositions. Through the events and moment-to-moment interactions storylines are created. These storylines allow a researcher to document the exact moment a student displayed a disposition or in our case, an emotion.

Methods

The participants of the study were two students using the pseudonyms Luna and Georgia. Luna was 10 and in Grade 4 and Georgia was 11 and in Grade 5. The study took place on a university campus close to their elementary school in the Midwestern United States. The two students participated in three semi-structured, task-based interviews (Goldin, 2000). The interviews took place over three weeks and each interview lasted approximately 60 minutes. For the three interviews, the students explored an unsolved graph theory problem, the Graceful Tree Conjecture. This is a problem that has been used in previous studies that have documented students display productive struggle while engaged with the Graceful Tree Conjecture.

Graceful Tree Conjecture

During the three task-based interviews, the students explored the Graceful Tree Conjecture. The Graceful Tree Conjecture is an unsolved problem in graph theory that is accessible to young children. While the parts of the problem the students explored have been previously solved, the entire problem remains unsolved. The students explored different classes of tree graphs which are connected graphs without a cycle. This means all tree graphs contain one less edge than node (vertex). It is believed that all tree graphs can be assigned a graceful labeling. This means when the nodes are distinctly labeled 1 through *n* or the number of nodes in the graph. The edges are labeled with the absolute value of the difference between the two connecting nodes. If the edges are labeled distinctly 1 through *n*-1, the graph is labeled gracefully (see Figure 1 for an example of a graceful and non-graceful labeling). The Graceful Tree Conjecture states that all tree graphs can be labeled gracefully.

Figure 1: Example of Graceful Labeling and Non-Graceful Labeling

Overview of Task-Based Interviews

During the three task-based interviews, Luna and Georgia explored different classes of tree graphs in increasing sophistication (see Figure 2). We not only challenged the students to find a graceful labeling for the first four distinct graphs but to find a pattern or justification that would show they could label any graph in the given class gracefully.

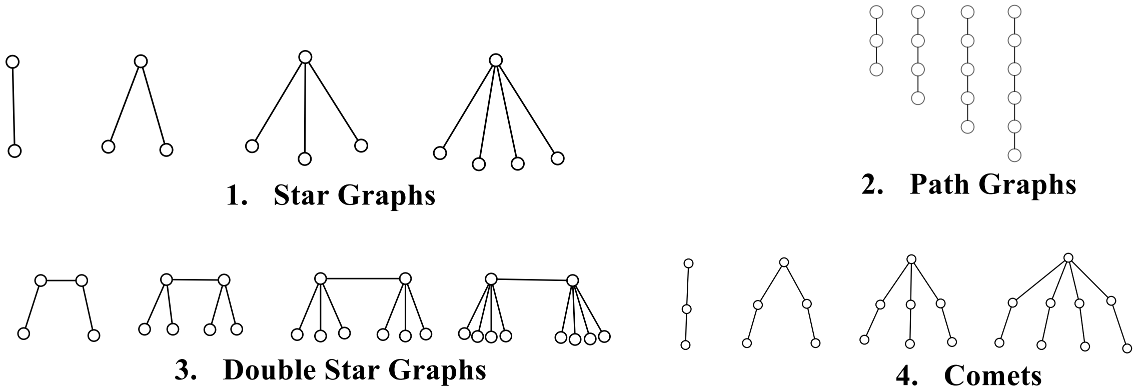

Figure 2: Graphs Students Explored in Increasing Sophistication

While the students explored the different classes of tree graphs, they were given a page on which to record labels for the first four distinct graphs in the class, a question that asked them to draw the next graph in the class, and statement to record their pattern. They were also given enlarged copies of each graph with numbered square and circle chips (see Figure 3 for an example of the students using the enlarged copy of each graph and numbered chips). The enlarged copies were given to the students so they could find solutions to the graphs without having to erase.

Figure 3: Example of enlarged copy of the graph and chips and image of how students self-identified emotions

During the first interview, the students were introduced to the meaning of the word conjecture, tree graphs, edges and nodes, what a graceful labeling was, and the Graceful Tree Conjecture. Next, they explored Star Graphs and Path Graphs. During the second interview, the students were reminded of the task, reviewed their solutions for Path Graphs, and explored Double Star Graphs. During the final interview, the students reviewed the problem, the Double Star Graphs, and explored Comets.

During each of the three interviews, the students self-identified their emotions or feelings using the Wong-Baker Faces (see Figure 3). Under the happy face, the words extreme joy was listed (we refer to this as level 6) and under the most upset face the words extreme frustration was listed (we refer to this as level 1). Students were instructed to move a chip to whatever they were feeling. An alarm sounded every three minutes as a reminder to mark their feelings.

Analysis

The task-based interviews were video recorded and the student work was collected. Next, the three interviews were transcribed using the program Transana (Woods & Fassnacht, 2016). This included the non-verbal actions, such as hand or arm motions, facial expressions, and the Wong-Baker level the students self-identified as. We then used a framework created by Else-Quest et al. (2008) and adapted by O'Dell (2017) to examine the emotions of frustration and joy the students displayed. Because of O'Dell's (2017) prior findings that the most common emotions students displayed were frustration and joy, we only examined the transcripts and video for those emotions (see Table 1 for modified framework).

Table 1: Analytic Framework

Emotion	Definition	Example
Frustration/Distress	Disappointment, discontent, displeasure	I am stuck. Man, this is confusing.
Joy/Pleasure	Delight, amusement, pride	I got it!

We examined the transcripts using the analytic framework and documented anytime an emotion of joy or frustration was displayed using Transana (Woods & Fassnacht, 2016). If the study displayed several statements or emotions of joy in a row, each individual statement or motion was documented as its own occurrence. We also document every time a student changed their self-identified emotion on the Wong-Baker Faces. Both authors both did this and discussed any discrepancies until we both agreed. Next, we created reports through Transana to account for each emotion and self-identified level displayed by the student.

Results

During analysis, we examined the emotions of frustration and joy the students displayed while working (see Table 2) and what level on the Wong-Baker Scale they marked. First, we will share the overall instances of frustration and joy, next we will give a description of Georgia during the third session to show how her emotion was displayed with her self-identified emotions, and lastly, we will share how the students self-identified on the Wong-Baker Scale.

Frustration and Joy

After analysis of the data, we used Transana to run reports of the instances of frustration and joy. We found 51instances of frustration and 92 instances of joy (see Table 2 for instances).

Table 2: Frustration and Joy Displayed by Interview

	Interview 1		Interview 2		Interview 3		Total	
	F	J	F	J	F	J	F	J
Georgia	6	12	5	16	22	19	33	47
Luna	2	10	13	18	3	17	18	45

Joy was typically displayed by both students when they made progress on the Graceful Tree Conjecture. Luna displayed joy, most commonly by verbal statements such as "I got it! So, I just have to figure out this pattern" when she found a graceful labeling. Another example of joy for Luna was, "that would be three. Yay!" with a big smile and saying "Okay, I have got to make sure I did this right." She also displayed joy when she was sharing her pattern verbally to one of the researchers and Georgia.

Georgia also displayed joy when she found a graceful labeling. Her joy was displayed more often by physical movement. One example is when she found a solution to a graph, she circled her arms around her head smiling and saying "my brain powers." Another instance of joy she motioned her hands over her page and said: "Ta-da!" She often clapped her hands with excitement, danced in her seat, or made a fist of joy.

Frustration was typically displayed while attempting to find a graceful labeling for a specific tree graph, searching for a pattern in a class of tree graphs, and when we pushed the students to create a generalization for each specific class of tree graphs. Luna gave verbal statements in a similar way that she displayed joy. She often things such as, "That wouldn't work" or "Wait, that wouldn't work because the five is supposed to be there."

Georgia again displayed her frustration more visually and kinesthetically. She would make faces of frustration, show frustration through her eye movements, leaning her body back in the chair, and toss chips down on the table when she got stuck.

Descriptive Case Study of Georgia

Georgia was a Grade 5 student. Her story from the third interview was chosen to be documented because she displayed the largest amount of frustration and joy. She worked on the Comet Graphs (see Figure 3). She began the session by identifying herself as a level five on the Wong-Baker Scale. Showing her joy of starting the interview session. The researcher then introduced the class of tree graphs called Comets. Georgia displayed joy by stating, "Those look really cool." She began to attempt to label the first distinct graph in the class (see Figure 4 for her labelings) and displayed several signs of frustration. After making an angry face at the camera, she said (time is shown as minutes and seconds into the interview):

(6:15) I am trying to remember my line graph pattern.

She then continued to move chips around. Several seconds later she made a motion with her hands and gave a big smile showing joy. She asked:

(6:40) Is that graceful?

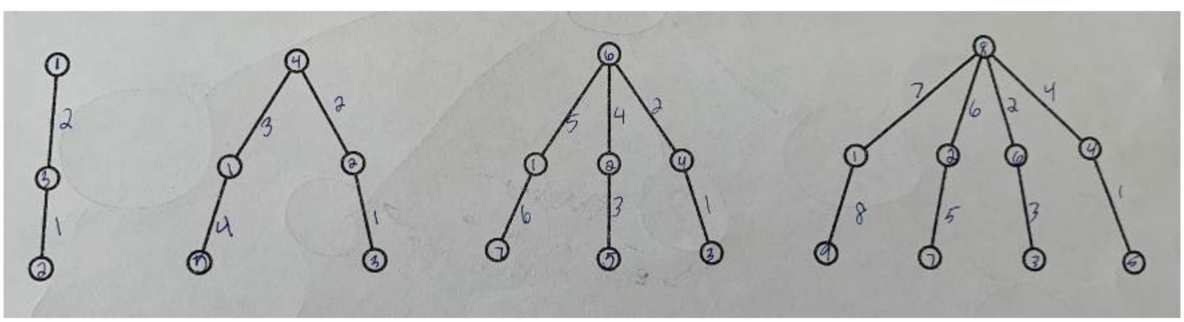

Figure 4: Georgia's solutions to the first four distinct comet graphs

After deciding that it is and recording her solution, Georgia began to work on a graceful labeling for the next graph. The timer sounded and she moved down to level four on the Wong-Baker Scale. Georgia stated:

(7:27) I am feeling a little confused.

She continued to work and at nine minutes into the interview, she shook her head displaying clear signs of frustration. Georgia stated with a frustrated undertone:

(9:05) Wait, how would you label it gracefully? Like, how would you?

Georgia continued to work on the labeling and thirty seconds later she gave a sly smile and said:

(10:15) Got it!
(10:21) That is graceful right?

The researcher told her she was correct and the three-minute timer sounded. Georgia moved her tile to level 5 or second highest level on the Wong-Baker Scale and stated:

(10:31) I feel good!

Georgia then began working on the third distinct graph in the class. The next timer went off and Georgia moved down to level 4. She continued to work. At 15:23, Georgia tossed a chip with an angry look on her face showing a sign of frustration. Next, she said:

(15:23) I am stuck.

She continued to work making two more facial movements of frustration and several seconds later said:

(16:25) Ugg

The timer went off again, Georgia moved down to level 3. She said:

(17:12) This is confusing.
(17:31) So I was just doing it line by line trying to figure it out.

Georgia continued to move chips around looking for a pattern and making visual signs of frustration but no verbal signs. At twenty minutes in, the researcher told both girls they stuck at the same place. Georgia put a big smile on her face showing a sign of joy. She continued to work on finding a label but did not show any signs of frustration. The timer went off at 24 minutes and Georgia moved her level up to four.

At 26 minutes in, Luna found a graceful labeling for the third graph. Georgia examined Luna's paper, followed suit quickly, and stated she found a solution that was the same as Luna's solution. Both girls began working on the next graph, repeatedly saying, "I think I can" and laughing. Several minutes later, Georgia stated with frustration:

(37:16) Urg, this is where I got confused last time.

When the timer went off next, Georgia moved her level down to 3. She continued to work on her graph. She stated:

(38:41) Oh, wait, no, no, no. Four minus what equals five. Four. Five. Six
(39:03) So I want this to be four so two minus what equals four?
(39:16) Ten. Six.
(39:40) And I want my three to be here. Brain is working.

At 39:51, Georgia began clapping and smiling and then stated:

(39:52) This is my only hiccup. That is what my teacher says. That is my only hiccup.

She thought she had an almost graceful labeling but had one edge that did not have the proper number for a graceful label (she was missing a five). She continued:

(40:18) I know but, wait, wait, wait.

Georgia made an angry face and made a joke about not liking fives. The timer went off and Georgia moved her tile up to four and continued to move chips around. Two minutes later, Georgia exclaimed:

(42:19) Eight! Ah. One is always here and then it goes. Wooo (hands on her head). That is the second biggest number because that is the third biggest number.

She continued to work. Next, she said excitedly:

(43:19) Oh, I am so excited!

She found a solution to the fourth graph and after prompting from Luna, Georgia moved her tile to level six but said:

(43:45) It is kind of confusing because I have mixed emotions.

And then she moved her tile back down to level four. She examined her solutions looking for a pattern and stated:

(44:23) So this is one and then and then it counts by twos. This is the second biggest number. That is the biggest. That is the third biggest number. This is the fifth biggest number.

Georgia continued to examine her solutions and found a pattern to generalize that any Comet graph can be labeled gracefully (see Figure 5 for her generalization). She shared that the bottom row of nodes were odd numbers and the biggest, the middle row of nodes was one and the even numbers, and the center node was the second biggest number.

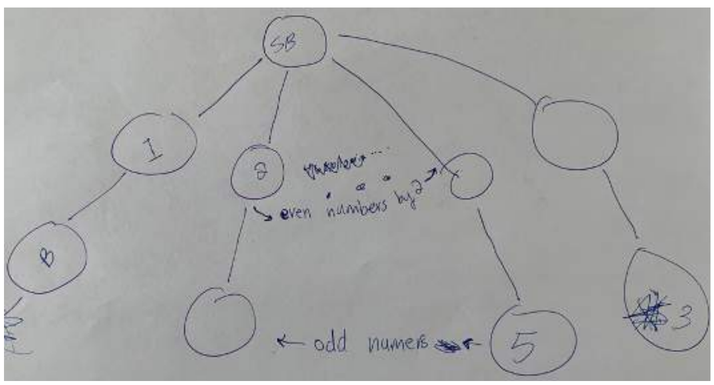

Figure 5: Georgia's generalization to Comet Graphs

Self-Identified Emotions

While the two students were engaged in the task-based interviews they self-identified their emotions of frustration and joy using the Wong-Baker Scale. The face of the most frustration we assigned a numeric value of one and the face with the most joy has a value of 6. Both subjects expressed familiarity with the scale from visiting the doctor's office. A timer went off every three minutes to remind them to mark how they were feeling and they were encouraged to move the tile to a new face if their emotion changed. We used Transana to run reports for every time the students self-identified using the tile and the Wong-Baker Scale (see Table 3)

Table 3: Frequency of tile movement on the Wong-Baker Scale

Level	Luna				Georgia			
	Day 1	Day 2	Day 3	Total	Day 1	Day 2	Day 3	Total
1	0	0	0	0	1	0	0	1
2	0	0	0	0	0	0	0	0
3	1	3	2	6	1	1	3	5
4	2	4	5	11	4	3	5	12
5	3	5	3	11	6	3	3	12
6	1	0	1	2	3	0	1	4

When either Luna or Georgia changed their tile to a higher level of frustration (lower number) on the Wong-Baker Scale they typically made some type of comment to discuss why they were moving down. The comment almost always had to do with them being "confused" or as Luna would say, "cornfused." Neither student ever identified being more frustrated than a level 3 besides Georgia on the first day. This happened at the end of the session and with the tile movement Georgia stated, "I feel sad because we have to leave."

Midway through the first interview, Luna decided that they should treat level six (extreme joy) as if they are experiencing similar feeling to riding a roller coaster and only move to that level if they felt that way. After that statement, there were only two more instances of extreme joy. Luna's extreme joy came at the beginning of the third interview and said, "I feel good."

Luna and Georgia tended to move their tile up when they were finding success on the graphs but that was not always the case. Several times both students would shift their tile higher when they reached a similar place on finding graceful labelings and were encouraged by the researcher to work together. Luna also tended to move her tile up at times while engaged in struggle and showing frustration but would not give a reason. For example, while trying to find a labeling for the third distinct Comet Graph she stated, while moving her tile up from three to four, "I am feeling better because" but did not give a reason. Both students never stated they reached extreme frustration besides, Georgia when she was sad they had to leave or even the level two. They also seldom reached level six of extreme joy.

Discussion and Conclusions

The results of this study are similar to Else-Quest et al. (2008) and O'Dell (2017) that students displayed both frustration and joy while engaged in mathematical problem solving. When O'Dell (2017) completed a similar study, she found students displayed more frustration than joy; however, we found the students to display more joy than frustration. Both studies still contained the oscillation between frustration and joy while engaged in problem solving. While Georgia and Luna were struggling through finding a graceful labeling they displayed several instances of frustration and when they found a successful labeling they displayed instances of joy. Other times they found joy in working together on the problem and through making jokes, such as "I think I can" while working.

When self-identifying their emotions, the two students repeatedly moved their tile to more frustrated when they stated they were "confused" and moved their tile toward joy when they found solutions. At other times we were not sure why they moved their tile toward joy while they were clearly still frustrated during the problem solving. Interestingly, they never—with the exception of Georgia being sad about the interview was over—moved to high frustration, level one or two. They only documented level three eleven times even though we documented 51 instances of frustration. The ratio of joy to frustration (levels 1-3 frustration and 4-6 joy) was 52 to 13. This demonstrated that even though the mathematical task was rigorous and an unsolved mathematics problem the students reported significantly more joy than frustration by a four to one ratio.

Overall, both of the students in this study were able to preserve through the struggle and frustration and were able to find joy and pride in their work on an unsolved graph theory problem. Unsolved problems are not typically included in elementary school but we found these problems to give students the opportunity to experience mathematics more similar to how mathematicians experience mathematics as a quest to describe patterns and relationships.

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