INVESTIGATING ELEMENTARY PRE-SERVICE TEACHERS' CONCEPTIONS OF MATHEMATICAL CREATIVITY

Anne N. Waswa
University of Georgia
anw04326@uga.edu

Kevin C. Moore
University of Georgia
kvcmoore@uga.edu

Research in mathematics education has overlooked creativity in mathematics, partially because of a lack of an accepted definition of mathematical creativity. The present study investigates elementary pre-service teachers' (PSTs') conceptions of creativity in teaching and learning mathematics. Data were collected using observations and semi-structured interviews with nine PSTs and analyzed using thematic analysis. PSTs' conceptions of mathematical creativity included using multiple approaches to solve problems, designing mathematical tasks from scratch, making learning challenging but not impossible, and exercising independence in learning. Implications of these results are applicable to teacher preparation programs, and they suggest a need for more research on the nature of experience(s) that shapes PSTs' conceptions of mathematical creativity and how to develop it.

Keywords: Instructional activities and practices, Affect, Emotion, Beliefs, and Attitudes.

Mathematics educators have argued that "mathematics is a creative, everyday human activity that cannot be built exclusively on rules and routines" (Schram, 1988, p. 8). Yet, according to Silver (1997), cited in Lithner (2008), "students hardly experience mathematics as the highly creative domain it is" (p. 7). If we intend to support students to discover and grow their mathematical talent, a change in broader classroom practices and curriculum materials is necessary, and in order to yield results from this change, creativity in mathematics should be part of educational experience (Mann, 2006). This educational experience should not be limited to students but should be made accessible to teachers who play a critical role in shaping the educational experience of students. Creative teachers are crucial to the development of mathematical creativity in each student through school mathematics education (Lev-Zamir & Leikin, 2011).

For us to nurture teaching with and for creativity by teachers, we must understand their conceptions of creativity (Lev-Zamir & Leikin, 2011). We address this need in the present study by investigating elementary pre-service teachers' (PSTs') conceptions of mathematical creativity. The purpose of the study was to understand how PSTs conceive creativity in the teaching and learning of mathematics. In the following sections, we review related literature and discuss a theoretical perspective that guided the study. We then describe the research methods, results and implications of the findings.

Literature Review

Research in mathematics education has overlooked creativity in mathematics (Haavold, 2018; Haylock, 1987; Leikin, 2009, 2011), partially because of a lack of an accepted definition of mathematical creativity (Mann, 2006). Among the researchers who have studied teachers' conception of creativity specific to mathematics, Bolden, Harries, and Newton's (2010) characterized elementary PSTs as holding a narrow conception of creativity in mathematics. The PSTs' conceptions were largely associated with the use of resources and technology and was bound up with the idea of teaching creatively instead of teaching for creativity. The National Advisory Committee on Creative and Cultural Education (NACCCE) (1999) defined teaching creatively as "teachers using imaginative approaches to make learning more interesting, exciting and effective" (p. 102), and they defined teaching for creativity as teaching that is aimed at developing students' creative thinking and behavior. The former emphasizes creativity in terms of teacher actions, while the latter emphasizes creativity in terms of student reasoning. Bolden et al. (2010) cited literature which indicated that

In: Sacristán, A.I., Cortés-Zavala, J.C. & Ruiz-Arias, P.M. (Eds.). (2020). *Mathematics Education Across Cultures: Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Mexico. Cinvestav / AMIUTEM / PME-NA. https:/doi.org/10.51272/pmena.42.2020

teachers of younger children believe that mathematics is creative but a closer investigation into their beliefs showed that creativity was viewed less in terms of the mathematics itself and more in terms of the creative activities such as construction, art, songs and rhyme, which are availed by mathematics sessions. Complicating the matter, Beghetto's (2007) study with middle and secondary PSTs identified that these teachers can view unique student responses as a potential distraction to classroom teaching, while PSTs from other subjects viewed such responses as worth pursuing. The potential effect of dismissing unique responses by students was that it can hinder the development of creative thinking, even if the teacher is using techniques associated with teaching creatively.

With respect to research investigating PSTs and their awareness of mathematical creativity, Shriki (2010) described the experiences of PSTs in a methods course focused on middle-school geometry where they engaged in activities aimed at cultivating their awareness of mathematical creativity and the complexity of the nature of creativity. She examined creativity by focusing on the value of the process or the product with citations from other researchers. As a process, creativity refers to cognitive abilities, conceptual thinking that involve flexibility, fluency, and originality, and non-algorithmic thinking. As a product, creativity is defined in terms of the novelty or uniqueness of a solution to a problem. Shriki argued that the learning environment and the nature of the assignments were relevant in aiding and growing PSTs' awareness of mathematical creativity and its multifaceted nature. She specifically illustrated that the learning environment provided PSTs with the freedom to work and design their own problems without having to follow certain rules or algorithms, and without fear of having a right or wrong answer. This, in turn, led to PSTs' development of intrinsic motivation, interest and curiosity. PSTs were also encouraged to be reflective about their insights and determine possibilities of generalizing the ideas, which in the end enhanced their mathematical knowledge.

Theoretical Perspectives

Researchers have approached creativity from different perspectives, and there is no general accepted definition of creativity (Haylock, 1987; Mann 2006; Sriraman; 2005). However, most researchers (Haylock, 1987; Lev-Zamir & Leikin, 2011; Leikin, Subotnik, Pitta-Pantazi, Singer & Pelczer, 2013) have adopted Guilford's (1967) characterization of the nature of creative thinking. The common features include fluency, flexibility, and elaboration, all of which fall within the divergent production ability of creative thinking. Fluency pertains to "a matter of retrieval of information from one's memory store," flexibility is "a matter of transformations of information," and elaboration is "a matter of producing implications" (Guilford, 1967, p. 11). Originality is another component of general creativity which Lev-Zamir and Leikin, (2011) defined as characterized by a unique way of thinking and unique products of a mental or artistic activity" (p. 19). These characteristics are mutually related, but they are not required to be present at the same time in order to claim the occurrence of creativity (Lev-Zamir & Leikin, 2011).

At a finer level, some differences exist in researchers' approaches to creativity. Piirto (1999), cited in Lev-Zamir and Leikin, (2011), distinguished between general and specific creativity. He identified general creativity with the application of problem-solving skills used in one field to solve problems in another field, and he connected specific creativity to the logical deductive nature of a particular field. Our study focused on mathematical creativity, which is a specific type of creativity that focuses on mathematics.

To label a behavior as mathematically creative, Haylock (1987) argued that both mathematics and creativity must be clearly present. This implies that for any process or product to be labeled as mathematically creative, it should be valid to the mathematics that was involved in that specific context. Lev-Zamir and Leikin's (2011) later added that defining mathematical creativity in the context of teaching should allude to mathematics, teaching, learning and creativity. An understanding of mathematical creativity was therefore important in this study.

For the present study, we adopt a model for creativity designed by Lev-Zamir and Leikin (2011) to characterize teachers' conceptions of creativity in teaching mathematics. In the model, teacher conceptions are explained in terms of teachers' mathematical content conceptions, which are how teachers view creative mathematical content, and teachers' pedagogical conceptions, which are "their awareness of didactic and psychological aspects of creativity in teaching and learning mathematics" (p. 19). Of the four characteristics of general creativity mentioned previously, their model focuses on flexibility, originality and elaboration because these three are unique to creative teaching. With respect to fluency, the authors consider it a primary indicator of how a teacher is qualified in terms of knowledge and proficiency, rendering it trivial in this model.

In Lev-Zamir and Leikin's model, teacher conceptions of creativity in teaching mathematics are further subcategorized as teacher-directed and student-directed under each of the three components: flexibility, originality and elaboration. Teacher-directed conceptions of creativity are actions by teachers that make them creative and these can be of a mathematical or pedagogical kind. Student-directed conceptions of creativity entail "connecting creativity in teaching mathematics with opportunities provided for the development of students' creativity" (p. 28).

Methods

Context and Participants

The study took place at a university in the southern United States. We recruited nine female pre-service elementary teachers from the early childhood education program, and their participation was voluntary. At the time of study, they were taking a mathematics methods course from either of the two sections taught by two different professors. One of the authors acted as a teacher assistant in both sections. The course was accompanied by a field experience component and it was the first of two courses that students take in the program.

We chose to focus on PSTs at the elementary level because this a critical stage of a child's mathematical development and how teachers are prepared to support them in this development is important. Mathematical concepts are interconnected and having a strong foundation for basic concepts in mathematics is likely to enhance understanding and creativity in learning as students progress to higher levels. We chose this specific course because of the nature of questions that PSTs explored throughout the course. PSTs were expected to reflect more on what mathematics is and what it means to know and do mathematics. These reflections can influence PSTs' beliefs about mathematics, which they are likely to carry on into their teaching together with the experiences they get from the methods course, ultimately impacting how they teach by shaping the approach and attitude of students in mathematics.

The nine participants came from different backgrounds in terms of race which in turn implies different cultures. Three out of the nine participants were PSTs of color and six were white. Of the three PSTs of color, two were born outside the US and moved into the US in their early age. The other was born and raised in the US. It was important to mention this variation in participants as their experiences are likely to inform their conception of mathematical creativity. Moreover, Leikin et al. (2013) indicated that some variables concerned with mathematical creativity depend on culture while other variables are intercultural.

Data Collection

We conducted two observations, one in each of the course sections for a duration of approximately 1.25 hours each. "A major purpose of observation is to see firsthand what is going on rather than simply assume we know" (Patton, 2015, p. 331). Having been familiar with the site, this was a statement that guided our observation in order to avoid uninformed conclusions. Given one of the authors' role in the classroom as a teacher assistant, it was her intention to avoid as much interaction as possible in order to capture majority of the events of the lessons and we therefore assumed the role of an onlooker (Patton, 2015) for the most part of the observations.

We also conducted one individual semi-structured interview (Roulston, 2010) with the nine participants. The semi-structured nature of the interview allowed us to deviate from the order of interview questions, because our interviewees' responses informed the choice and order of questions. For example, we did not have to ask all questions that we had in our protocol when an interviewee responded by also answering a follow-up question(s). We also used probing questions to follow-up on our interviewee's responses and yield more detail and explanations about what our interviewees had said (Roulston, 2010). While probing, in most cases, we used our interviewees' own words to formulate questions. The interview focused on PSTs conceptions of what it means to teach mathematics creatively followed by their conceptions of what it means to learn mathematics creatively.

Data Analysis

To analyze data, we used thematic analysis (Braun & Clarke, 2006) which is a "method for identifying, analyzing, and reporting patterns (themes) within data" (p. 79) or a process that involves looking for patterns within the data and categorizing those patterns according to themes (Fereday & Muir-Cochrane, 2006) cited in Bowen (2009). Thematic analysis uses coding as a strategy (deMarrais & Lapan, 2003). The themes were mainly from the aforementioned framework of Lev-Zamir and Leikin (2011). We analyzed our participants' responses to interview questions and occurrences from the classroom observations in light of flexibility, originality and elaboration. From these categories, we further grouped our findings into the subgroups of teacher-directed and student-directed conceptions of mathematical creativity. Data in the flexibility group included statements about different types of transformation of information in teaching and learning mathematics and varied solution paths to problems that could result from the teacher and/or the student. Data in the originality group entailed PSTs' statements about novel ways of thinking while teaching and learning. Novelty in this case referred to uniqueness from the usual accepted norms and conventions of problem solving in the process of teaching and learning. Data in the elaboration group constituted PSTs' statements about advancing thinking to higher and related levels.

Results

The data presented in this report are from three (Nelly, Paula, and Laura – pseudonyms) out of the nine PSTs. We chose to focus on these three participants here because they provided concise but representative data of the nine participants. We categorized the findings in terms of teacher-directed conceptions of creativity and student-directed conceptions of creativity as explained in the theoretical perspective.

Teacher-Directed Conceptions of Mathematical Creativity

This form of creativity included PSTs' views and actions, both mathematical and pedagogical, that enhance their teaching of mathematics creatively. The different views of PSTs' teacher-directed conceptions of creativity included using multiple approaches to solve problems, designing mathematical tasks from scratch, and making learning challenging but not impossible. Table 1 represents a summary of these conceptions, their description, and their perceived enactment, or the actions that teachers envision to ordain the conceptions.

Table 1: PSTs' Conceptions of Mathematical Creativity in Teaching

Conception	Description	Perceived Enactment
Teacher-Directed Conceptions of Mathematical Creativity		
Use multiple approaches to solve problems	Finding the solution to a mathematical problem using varied solution paths.	Provide support for students, both pedagogical and mathematical to help them think of and use various perspectives when solving problems. This support can be in terms of using supporting and extending moves, and purposeful questioning.
Design mathematical tasks from scratch	Exercise teachers' independence and creativity in determining content and context that will be accessible to students.	Devise contextual problems, use manipulatives, hands-on activities, and games that are appropriate to the goals of the lesson to engage students in thinking about mathematical concepts creatively. Integrate other subjects e.g. English and Science in mathematics lessons.
Make learning challenging but not impossible	Providing challenging problems and situations that will build students' intellectual curiosity and challenge them to think deeply about the problem and its solution.	Teach concepts to enhance sense making by students by not dwelling on algorithms. Extend students' thinking through questioning. Use purposeful questioning to elicit ideas that will help students think for themselves with less input from the teacher. Encourage productive struggle.
Student-Directed Conceptions of Mathematical Creativity		
Exercise independence in learning	Students' ability to develop their own perspective into learning and reasoning independently.	Students solving problems in their own way without being directed on how to do everything. Students putting new perspectives in problem solving.

Use Multiple Approaches to Solve Problems. This conception involved a teacher believing mathematical creativity as teachers' ability to provide support for students, both pedagogical and mathematical, to help them consider multiple perspectives when solving problems. Example quotes from the participants are:

Paula: I think it is super important that kids have different tools coz I don't think people my age and adults were really given anything other than the standard algorithm to solve a problem.

Laura: I think the biggest thing is learning all these different kinds of strategies and knowing that you should encourage it for kids because I feel like usually teachers just want you to stick to a specific strategy.

Both Paula and Laura identified the relationship between a variety of strategies, knowing and flexibility. They emphasized the need to incorporate different approaches to solving problems and making them accessible to students by not restricting them to a specific approach. More generally, their idea fits under the flexibility component of mathematical creativity because shifting perspectives in problem-solving can be considered as a form of transformation of information, say for instance, multiple representations (e.g. visual, symbolic) of a solution.

Design Mathematical Tasks from Scratch. At the time of study, PSTs were working with elementary students once a week during which they designed activities and problems to be worked on by their students, an exercise that they deemed as involving mathematical creativity. Example quotes from the participants are:

Nelly: Making up problems and, like the literature assignment, I felt like that was really creative… We pretend to be elementary schoolers a lot. And so we had to put ourselves in their mindset…. But including games that kids have to problem solve, like problem solving, I think is more, like introduces more creativity into math."

Laura: So I think we should make math more creative for them (students). So they can still be learning something while, doing a fun activity. Maybe being able to work in groups or making games out of the math,…

Nelly and Laura viewed mathematical creativity as generating tasks that encourage activity while learning. Specifically, they mentioned the need to incorporate games in activities. Creating such activities required them to position themselves like students in order to ensure accessibility of the materials that they would generate.

During classroom observations, PSTs were challenged to make in the moment decisions on how they would support their students' thinking in their placement. Their professors brought up hypothetical scenarios, for example, on student misconceptions and asked PSTs to think of how they would support and/or extend their students' thinking. During our interactions with the PSTs, they also identified the need to integrate science and English in their lesson, what Nelly explained as talk about math not in a math class as being mathematically creative. Generally, we categorized this conception under originality trait of mathematical creativity because the process included considering student's level of understanding and designing tasks that would be accessible to them in terms of context and content, hence requiring specificity and novelty in thinking about the nature of the activities.

Make Learning Challenging but not Impossible. PSTs explained that it is important to put less pressure on students but at the same time maintain their interest and engagement in learning. The following example quotes supported this finding:

Nelly: creative learning should feel more fun and more challenging but not impossible… teachers should let them (students) figure out things for themselves, instead of just telling."

Laura: But I can see like, why it's better to be more creative because you can put in a whole lot of different perspectives into it. And it doesn't have to be so straightforward.

Nelly emphasized the need for teachers to allow students to make sense of mathematical ideas on their own and Laura also supported this idea by emphasizing the idea of not being straight forward and having students develop their own perspectives into learning. We categorized this conception under the originality component because we portrayed making sense of mathematical concept with less scaffolding as requiring a higher personal cognitive input.

Student-Directed Conceptions of Creativity

PSTs' student-directed conceptions of creativity were closely aligned with their teacher-directed conceptions of creativity. They included students generating their own solutions to mathematical tasks, which we termed as exercising independence in learning.

Exercising Independence in Learning. PSTs viewed students' ability to generate their own and varied solutions and explain their thinking as an indication of mathematical creativity. Example quotes include:

Nelly: students should be able to do it (math) their own way and not being told exactly how to do everything.

Paula: I would say like being able to come up with like, explain it back to me verbally,…

Both Nelly and Paula address the need for students to own their learning with less input from the teacher and by describing their reasoning. They are allowed the freedom to bring their own perspective into learning and make sense in a manner that best suits their way of making sense. We situate this conception under originality because doing it in your own way and explaining it verbally foster uniqueness in individual thought processes.

Discussion and Conclusion

Findings of the present study indicate that PSTs conceive teaching mathematics creatively as supporting students to use various strategies to solve problems when teaching, developing appropriate mathematical tasks to enhance student activity and engagement while learning, and providing challenging but accessible learning opportunities for students. PSTs conceived learning mathematics creatively as students being independent in learning. These conceptions, teacher-directed and student-directed are intertwined in that, perceived enactments of teacher-directed conceptions of mathematical creativity enhance student-directed conceptions of creativity. For example, when teachers use purposeful questioning to elicit ideas that will help students to think independently and when they support students to use multiple problem solving approaches, teachers can enhance student independence in learning by challenging them to think and bring in self-generated and new perspectives. This observation is partly in line with Lev-Zamir and Leikin's (2011) assertion that defining mathematical creativity in the context of teaching should allude to mathematics, teaching, learning and creativity. Our reason for using partly in line with is because we observed that PSTs commented on, for example, using games and manipulatives to make the learning of mathematics fun, and as an avenue to teaching mathematics creatively. However, the math behind or within the "fun" was not given keen attention. PSTs tended to overlook creativity and mathematics, which was emphasized by Haylock (1987) and Lev-Zamir and Leikin (2011) and attended more to teaching and learning in their descriptions of their conceptions of mathematical creativity. Care should therefore be taken to differentiate between teaching creatively and teaching for creativity (NACCCE, 1999) to ensure that students do not just have fun in class while engaging in games but that they also understand the math behind or within the fun and develop their own creativity.

Connecting to research, results of this study are not unique to mathematical creativity, and thus tie closely to other findings and recommendations from researchers whose focus is not necessarily on mathematical creativity. To begin with, encouraging students to use different approaches to solving problems does not occur naturally if a teacher has not anticipated some of the strategies that students are likely to use. Anticipating student strategies is a key practice for successful orchestration of classroom discussions. Discussions stimulate interaction, an important catalyst to creativity, as the teacher responds to students using assessing and advancing questions, and notices important aspects of student thinking during instruction (Smith & Sherin, 2019). Second, teachers who are open to designing tasks from scratch are likely to demand of the same from their students, by not just providing students with problems to work on, but also challenging them to generate problems that address specific mathematical concepts. This practice demands high cognition and is at the level of doing mathematics (Smith & Stein, 1998). It discourages algorithmic thinking, requires students to comprehend mathematical concepts, processes, and relationships, and demands self-monitoring, only to mention but a few, according to Smith and Stein (1998). These conditions are equally important to cultivating mathematical creativity for both teachers and students. Third, providing students with challenging but not impossible questions has a potential to stimulate students' intellectual curiosity and hence develop their creativity. The points we raise in this discussion do not mean that we should not pay close attention to creativity in mathematics in the field of mathematics education, but rather consider investing in its research as it is a potential contributor to the growth and improvement of teaching and learning of mathematics.

We note that the findings of this study are not a complete representation of what conceptions of mathematical creativity are. Conceptions are informed not just by educational experiences, but by culture and beliefs and therefore this area of research is open to more studies, particularly with a focus on specific types of experiences that shape these conceptions. This will be important in shaping our teacher preparation programs to provide PSTs with those experiences necessary to support their understanding and development of mathematical creativity and that of their students.

We close with noting that a major observation from our literature review was that research in mathematics education has overlooked mathematical creativity due to lack of an accepted definition of mathematical creativity. We argue that we cannot overlook mathematical creativity simply because of lack of coherence in existing definitions of what mathematical creativity is, but we can instead focus on specific conceptions of it and develop its understanding because by doing this, we will not only be focusing on what it is, but also on what it could be, which is not always done in other fields of research with agreed definition of constructs. By doing that we won't limit our understanding on what it is but can explore the what it could be and find connections that will expand our horizon in understanding of the concepts, and possibly impact the field of mathematics education.

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