"DYSLEXIA IS NATURALLY COMMUTATIVE": INSIDER ACCOUNTS OF DYSLEXIA FROM RESEARCH MATHEMATICIANS

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Using neurodiversity as our theoretical framework, rather than a deficit or medical model, we analyze the narratives of five dyslexic research mathematicians to find common strengths and challenges for dyslexic thinkers at the highest level of mathematics. We report on 3 themes: 1) highly visual and intuitive ways of mathematical thinking, 2) pronounced issues with memorization of mathematical facts and procedures, and 3) resilience as a strength of dyslexia that matters in mathematics. We introduce the idea of Neurodiversity for Mathematics, a research agenda to better understand the strengths (as well as challenges) of neurodiverse individuals and to use that knowledge to design better mathematical learning experiences for all.

Keywords: Equity and Diversity; Advanced Mathematical Thinking; Geometry and Geometrical and Spatial Thinking; Representations and Visualizations

The neurodiversity movement, developed from activism of people with autism, dyslexia, and other cognitive differences, demands that such individuals be understood not as deficient, but as different: part of the natural and beneficial cognitive diversity of society (Robertson & Ne'eman, 2008). Cognitive neuroscience demonstrates evidence of both strengths and challenges for individuals with dyslexia, the largest group of students in the special education system. While some individuals with dyslexia have been highly successful in mathematics, students with LD/Dyslexic score on average lower than their neurotypical peers on mathematics achievement tests, with gaps widening over time (Wei, Lenz, & Blackorby, 2013). However, educational research has focused almost exclusively on identifying and remediating deficits of individuals with dyslexia, with a pronounced silence on how related strengths might matter for learning mathematics. Rejecting a deficit lens on the mathematical thinking of students with dyslexia/LD, we propose Neurodiversity for Mathematics, a research agenda to better understand the strengths (as well as challenges) of neurodiverse individuals and to use that knowledge to design better mathematical learning experiences for this large group of learners.

Using interviews and narrative analysis, this project investigated this issue from the perspective of *neurodiverse insiders* who have experienced learning mathematics with success at the highest levels. Using neurodiversity as our theoretical framework, rather than a deficit or medical model, we analyze the narratives of five dyslexic research mathematicians to find common strengths and challenges for dyslexic thinkers at the highest level of mathematics. We report on three themes: 1) highly visual and intuitive ways of mathematical thinking, 2) pronounced issues with memorization of mathematical facts and procedures, and 3) resilience as a strength of dyslexia that matters in mathematics.

We ground the proposed research in the academic field of Disability Studies (DS), which recognizes that although individuals have natural biological variations, it is the social effects of difference that disable rather than the impairments themselves (Linton 1998). From the DS perspective, the medical model and behaviorist tradition depict disability as deficits located within individuals resulting in identifying, pathologizing, and stigmatizing difference; thus requiring specialized knowledge (e.g., special educators) for individualized instruction and remedy. Academia continually reinscribes difference between children with and without disabilities in an unconscious effort to maintain the status quo, in which children with disabilities are conceptualized as fundamentally different from

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"normal children" (Linton 1998). In order to understand and improve the experiences of dyslexic students, we must ground our analysis in the perspectives of those with this difference. This study seeks to provide a new lens on the mathematical learning of neurodiverse individuals, grounded in the experiences of those with cognitive differences, rather than the perspectives of researchers who are often neurotypical. Our study is a collaboration between the first author, a cis white female former math teacher and special educator currently working in mathematics and disability studies at the university level, and the second author, a cis white male research mathematician with dyslexia.

Dyslexia is a hereditary neurobiological disability characterized by difficulties in reading, writing, and spelling, often unexpected in comparison to other academic skills (Lyons et al., 2003). While originally called dyslexia, these differences were reclassified "Learning Disabilities" when Specific Learning Disability became a category of special education services under US law. Learning Disabilities (LD) is a broader category that includes LD in the areas of reading (dyslexia), writing (dysgraphia), and mathematics (dyscalculia), as well as other variants of LD such as Auditory Processing. Individuals can experience LD in multiple areas. What tends to be consistent across LD is some form of processing and/or language difference that significant affects learning in school. A significant population of individuals with LD also have diagnoses of Attention Deficit Hyperactivity Disorder (ADHD). While LD is the term in US law, individuals often prefer the term "dyslexia." Currently, laws in the US are shifting back towards dyslexia, specifically towards advocacy for multi-sensory, systematic reading instruction. Much of the research in LD and math is focused on students with dyscalculia, or significant difficulty learning mathematics. However, students with LD in general, which is most often most pronounced in reading, significantly underperform in mathematics (Wei et al., 2013).

Currently, there is little overlap between research in mathematics education and special education mathematics research (Lambert & Tan, 2020). These research traditions are largely separate because of pronounced theoretical and methodological differences over their history of development as fields (Woodward 2004). Based on a recent analysis of the literature on mathematics learning across the two fields (Lambert & Tan, 2020), we found that special education primarily understands mathematics learning through behavioral and information processing approaches, using quantitative methods that focus on large populations. In contrast, mathematics education is focused on constructivist and sociocultural approaches to understanding mathematics learning through a focus on individual thinking and classroom contexts. Research methodologies in mathematics education include both quantitative and qualitative methodologies. Special education research frames the achievement gap for students with learning disabilities as a problem of cognitive deficits in individuals and seeks interventions to remediate deficiencies. Recommended interventions are primarily explicit or direct instruction. In previous work, we have critiqued this focus on pedagogies as deficit-based and promoting narratives about students with learning disabilities being unable to learn through inquiry (Lambert 2018). These narratives, which we face continually in schools, are antithetical to the larger goal of increasing access to higher-level mathematics for students with learning disabilities.

There has long been speculation about the connection between dyslexia and visual-spatial talents, dating back to Orton in 1925 (Schneps, Rose, & Fischer, 2007). There is evidence that people with dyslexia have strengths in visual-spatial thinking, although not conclusively. Some of the differences in findings can be attributed to different ways to define and assess visual-spatial thinking (von Károlyi & Winner, 2004). One strength associated with dyslexia in several research studies is 3-D spatial thinking, connected to strengths in mechanics and complex visualization (Attree, Turner, & Cowell, 2009). Another strength is interconnected reasoning; many individuals with dyslexia tend to make unique associations between concepts, focused on the big picture (Everatt, Weeks, & Brooks, 2008). Individuals with dyslexia describe using this strength to analyze large data sets and recognize patterns. Dyslexic students scored higher than nondyslexics for original thinking (Akhavan Tafti, Hameedy, & Mohammadi Baghal, 2009) and creativity for tasks requiring novelty (Everatt, Steffert, & Smythe, 1999). There is evidence that these strengths are neurologically interconnected with the challenges of individuals with dyslexia (Eide & Eide, 2011). This research has been strongly supported by an increasing movement of adults with dyslexia to reject the medical, deficit model of dyslexia.

In studies on successful dyslexic adults (2007), Rosalie Fink has found that while the adults had several developmental pathways to becoming successful adult readers, a consistent thread was the importance of reading in areas that individuals were passionate about. Another pattern in Fink's findings is that these successful adults continued to have significant early deficits in reading, difficulties with letter switching and decoding, yet were successful readers at a much higher level (both as self-reported and based on reading comprehension assessments). We know of only one study in the area of mathematics that is similar to our work; a collaboration between a mathematics education researcher and an individual with dyscalculia, investigating how the second author developed her own strategies to support learning in an undergraduate mathematics program (Lewis & Lynn, 2018). We believe that collaboration across difference is necessary to develop new understandings of the potential for dyslexic students in mathematics.

Research Questions

1. What have been the experiences and mathematical learning trajectory for individuals with dyslexia in higher mathematics?
2. How do individuals with dyslexia approach mathematics?

Methods

Participants

Participants were recruited using emails to professional organizations of research mathematicians by the second author. All participants were currently employed at universities in mathematics or related STEM departments. Four out of 5 participants reported being actively engaged in current mathematical research, with one participant focused on teaching. 3 out of the 5 participants were diagnosed with dyslexia and/or a reading learning disability during K-12 schooling. Another participant was given a diagnosis later in life related to the diagnosis of a child. Another participant had a diagnosis of a related disability (ADHD) but whose primary difficulty was reading and reading comprehension. Two out of the five participants reported a diagnosis of ADHD, one had significant speech and language delays as a child, and one participant self-identified as autistic. Two out of five participants identified as cis female, with 3 identifying as cis male, however we use the pronoun "they" throughout the document to avoid identification. Four out of 5 participants identified as white or Caucasian, with one Asian-American participant. We identify this lack of diversity as a significant limitation to our work and hope to expand the populations included in future studies. We report data on participants (P1-P5) in the aggregate to avoid identification, as not all participants were comfortable with disclosing their disabilities in the university setting. Thus, we purposefully report themes across the interviewees, rather than describing each individual as a case study. We also use the pronoun "they" to avoid identification of participants.

Data Collection

The first and second author together interviewed each participant for between 60 - 90 minutes. The interview was semi-structured with questions in the following categories: 1) description of current mathematical work, 2) school experiences in mathematics, 3) school experiences and diagnosis of dyslexia, 4) connections between dyslexia and mathematics. Interviews were video-recorded and then transcribed.

Data Analysis

We identified narratives within the interviews based on transcripts. We identified two kinds of narratives. The first was Life History Narratives, more typical retrospective narratives retelling a life event. We also identified a kind of narrative we called Narratives of Thinking, which we define as nontraditional narratives without a set sequence of events, with an experiential description of how it feels to think in a certain way, here related to mathematical thinking and problem solving (Lambert 2019). After identifying narratives, we coded thematically (Riessman 2007) with both apriori and emergent coding. All narratives were coded by the first author with discussion of coding categories with the second author.

There is a tension in our work between identifying commonalities between the participants and making diversity between them quite clear. When we report that 4 out of 5 interviewees noted their own strengths in visual thinking, we are not suggesting that 80% of people with dyslexia are visual thinkers. Our small set of studies is an opportunity to identify some common themes within a small, specific subgroup of the dyslexic population, dyslexic mathematicians. We would need different research to answer questions about how these preferences apply across people with dyslexia more broadly.

Findings

The research mathematicians were working in the following areas of mathematics with multiple participants in some categories: real analysis, three-dimensional geometry, topology, and algebraic topology. The life narratives of these research mathematicians describe a non-direct pathway to becoming a research mathematician. They describe barriers that could have limited their process, such as calculus focused on memorization, or classes such as organic chemistry focused on memorization. All participants noted that they moved forward in mathematics once they reached a place in which they were fascinated by the problems, most often, a visual-spatial set of problems to solve.

All participants described dyslexia as a set of strengths and challenges, although the specifics of those strengths and challenges varied between participants. P4 notes, “the dyslexia... I explain to people, it's sort of like you're strong in one thing, but it makes you weak in others.” Of course, our study attracted individuals who were interested in talking about their dyslexia, which possibly created a group that was more positive about dyslexia than a randomly constructed group. In this paper, we report on three themes: 1) highly visual and intuitive ways of mathematical thinking, 2) issues with memorization of mathematical facts and procedures, and 3) the development of resiliency as a strength.

Theme 1: Highly Visual and Intuitive Ways of Mathematical Thinking

Flexible, creative thinking. All participants noted that they had a history of unusual ways of solving mathematical problems. 4 out of 5 participants described visual thinking quite specifically. 4 out of 5 participants also noted that they were known for flexible, creative, “out of the box” solutions to complex problems. P3 shared that one of their collaborators once described them, saying,

> I talk in ghosts and mists. My brain seems to be really, really comfortable with just throwing out ideas. It just really is very flexible. It doesn't like boxes. It's just very, very flexible. And so, I get a sense that something is true, or something that I want, I need, is there. And then my brain really doesn't get bothered by the fact that some ideas don't work, it just will throw out lots and lots of ideas and sort of wander. And that drives co-authors nuts, because they'll say, "Oh, I see? That idea doesn't work." And it doesn't slow me down one bit. My brain just

> has like five other weird ideas, two of which you can throw out immediately, and the three others you have to spend time on. And it just sort of keeps working that way. (P3)

P1 noted their personal strengths in mathematics as:

> Coming out with the idea that pushes you beyond the routine. So thinking about things, especially a visual or spatial ideas. Questioning and poking at the routine to say how do we express this idea? So sort of coming up with ideas that are not in the routine, especially things that are related to images. (P1)

Visualization. Four out of 5 participants repeatedly brought up visualization as a personal strength in their own mathematical thinking. In multiple interviews, participants described a duality such as "verbal" vs. "visual", or "algebraic" vs. "geometric." Four out of 5 participants identified as visual and/or geometric mathematical thinkers, with P1 noting that they learn through "geometry first, thinking through space" and "I can do immensely technical work in images that others can do in language."

P4 describes a strong predilection for thinking visually, not just in mathematics but across topics; "Well my entire memory is sort of visual, it's like playing back little snippets of film." They became interested in "three-dimensional geometry and topology. Anything that I can draw or sculpt or anything that's like three dimensional and sort of visual-based." Once when struggling in a physics class, they saw a particular image in the text, of vector fields on a surface, and suddenly, the "pictures made it make sense." P4 prefers to not only think with visuals, but to write with them as well, noting a strong preference for storyboarding mathematical papers using a series of images. Another participant also indicated that they chose their mathematical topic based on their preference for visual thinking, specifically "picture drawing and the topology of it" and "I liked the fact that I could pin it down and think about it as something real" (P2). Another participant identified as both:

> A details kind of thinker and like a visual thinker. I can't get interested in the details unless I have the picture that I think I'm working out the details for. But, once I have a picture of what I think should be going on, then the details become interesting . . . It's not the other way around. (P5)

Symmetries and the commutative property. Three out of 5 participants at some point in their interviews suggested their dyslexia may have been connected to a way of seeing mirror images and/or symmetries in geometric shapes and algebraic equations, "Well I think I saw symmetries, I saw equations easily because of it. Because my brain would flip things around very easily. I understood equations quickly and easily because of my dyslexia" (P2). One participant wondered aloud if, "Somehow dyslexic thinking is naturally commutative?" (P1).

All participants noted difficulties with language in relationship to mathematics, each slightly differently. Some discussed primarily issues with communicating visual thinking through language, or through the more linear pathway of writing. One participant described how it was difficult for them to visualize symbols/mathematics when the only modality that is being used is talk.

> P5: This even happens when I'm with mathematician friends and they'll be vocalizing an argument. There's no white board and they'll say, you do this and then you ... Yeah, I'm not necessarily going to follow the point. But, I'll go back to my room later and I'll remember enough of the points that they were trying to make that I'll get it. And, I'm okay with that, I don't have to be as quick witted as some of my colleagues are in mathematics, and I don't mind that.
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> A: So, it's really different for you if there's paper, or there's a white board?
>
> P5: Yeah, if I can visualize things I'm much better off. Well, is that true? ... I'm pretty good at visualizing, but what I'm not good is transcribing spoken language into notation . . . If someone were to read out loud the definition of continuity for all blah, blah, blah. I would say, "Yeah, that might be right. That might be wrong. I have no idea." So, it's this translation between spoken

> language, and it's kind of linear notation that mathematicians tend to use. I'm not good at that part. I'm good at visualizing geometric things, but not visualizing notation (P5)

This suggests that strengths in visualization may be connected to challenges around translating across different forms of language and/or modalities, particularly from visual thinking into verbal language, or vice versa.

Issues with Memorization of Mathematical Facts and Procedures

None of the participants noted significant difficulties with mathematics in elementary school, with one consistent exception across participants: difficulty with the times tables and/or memorizing mathematical procedures. Some brought this up spontaneously, and for others, they clearly did not connect memorization of facts to mathematics. In this exchange with P2, the first author is asking whether or not they had any difficulties with math in K-12.

> A: Was there any part of math, like in elementary school, middle school, or high school that was challenging for you?
>
> P2: No.
>
> A: So memorization of facts was not challenging for you?
>
> P2: Oh I never could memorize anything. I had to derive everything . . . Yeah, I've never been good at memorizing things, just like I couldn't memorize how to spell words, I couldn't memorize facts in math. So I paid attention in class, and I had good enough teachers that they derived everything. And I figured out how to derive everything I needed to know, and I just derived everything I needed to know. You take a trig class, for instance, okay ... I know the trig identity for sin of alpha plus beta. From that trig identity, I can derive all the other ones. And then if I needed any of them, I would just do that. But I never actually like memorized them. I still don't memorize them.

Similarly, P1 noted that their mother taught the multiplication tables through a smaller set of memorized facts, specifically the squares, and then encouraging P1 to build equations through the distributive property from known facts.

In addition to the multiplication tables, participants noted the difficulty of any kind of memorization "without structure." P1 notes,

That is one of the reasons I'm slower. I have really good memory for connected facts. I can't remember phone numbers at all. Learning foreign languages was the one bit of school that I hated because you have this long list of words that had no connection to anything. So memorization without structure. So I memorized the structure. (P1)

Another participant noted that they had a history understanding "concepts" in mathematics and struggling with "the details." When we asked what they meant by details, they told a story about being negatively judged for their lack of memorization of the multiplication tables in elementary school:

> I could've explained to you with a picture why nine times five was 45, and my friends could tell you that it was 45 but they couldn't tell you why. And it struck me as really upsetting that someone that, just memorizing that number, was valued more than me understanding why that was the right answer. And it's always been a problem. But it just seems to me that why something is true is much more important than knowing that it is true. (P3)

Several participants noted that their difficulties with memorization were connected to the expectation of speed connected to memorization. Not only was memorization without structure very challenging, participants were asked to do this task under time pressure, which made it feel impossible.

Theme 3: Developing Strengths Through Struggle

Four out of 5 participants specifically mentioned resilience as a strength of dyslexia. More specifically, participants noted that working through challenges made them more resilient and perseverant, which became a considerable strength for them in higher mathematics. When asked what a strength of dyslexia is, P2 said, “Resiliency, I guess. Just being able to kind of overcome things that are not necessarily the easiest for you.” P5 described their “coping mechanisms”:

> Basically being comfortable with the fact that I'm not going to be fast at a lot of things. And, being okay with not being fast, that's really pretty important because I think that this kind of reading comprehension and fluency stuff. The fluency tests really make you think that speed is the whole deal. And, it was really important for me to sort of realize that no, that's not what matters. (P5)

Success in mathematics, participants noted, comes with hard work. Because math gets hard for almost everyone, understanding what to do when that happens is a gift for a mathematician. As P1 notes,

> Sort of actually everyone is facing struggles. Calculus is hard for most people. And so what we can understand about how people get through it when they have greater struggles is really useful for the people who are having smaller versions of those same struggles. And that notion of motivation ... The point that people who want to do this material but it's really hard for them can do it then surely that's a principle for all education. (P1)

Discussion

This research contributes by challenging the deficit-based approach typically used in educational research with students with dyslexia, thus potentially opening new avenues of educational research that move beyond the medical model of disability which locates LD/dyslexia solely in individual students. While this paper focused on only 3 themes, these findings suggest ways in which we can make math classrooms more accessible for students with dyslexia. We describe these as initial tenets of *Neurodiversity for Mathematics*, and plan future research to both explore insider perspectives further and to test these ideas in the classroom.

1. Offer opportunities for visual thinkers to learn new concepts through visual thinking. Not only provide multiple modalities for learning mathematics, but explicitly connect different kinds of representations. For example, one participant explained how their own mathematics teaching relies on visuals, but also with explicit connections to algebraic representations for those who preferred to think that way.
2. Remove the focus on memorization and procedural learning for students with dyslexia. As P1, who attended school outside of the US, noted;
   The high school calculus in the U.S., I think I wouldn't be a mathematician if I'd taken that because it's so emphasized it's fast, technical things ... And I make a lot of errors when doing calculation, and when you have a test which is multiple choice which is designed to map you into all your errors, I would have got very poor scores (P1).

When mathematics focuses on speed and memorization “without structure,” the potential of those with dyslexia will not be realized in our schools.

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