IDENTIFYING OPPORTUNITIES TO ENGAGE IN LITERACY PRACTICES: A FRAMEWORK FOR ANALYZING CURRICULUM MATERIALS

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In this report I have created an exploratory framework to identify opportunities to engage in literacy practices within mathematics curriculum materials. This framework describes "unstructured literacy opportunities" and "structured literacy opportunities" for each of the language modalities of reading, writing, speaking, and listening. Different structures within each modality are also detailed along with connections between the modalities. The framework is then applied to Illustrative Mathematics (IM) curriculum materials to reveal patterns of how different types of literacy opportunities are addressed and connected.

Keywords: Curriculum Analysis, Communication, Classroom Discourse, Instructional activities and practices

Written and spoken language serve important functions for communicating, receiving, and retaining information. Language also manifests in complex ways. Aguirre and Bunch (2012) describe how students must navigate a myriad of language demands in mathematics classrooms, differentiating such demands into modalities of reading, writing, speaking, and listening (along with a "representing" demand critical to mathematics in particular). The authors also describe how these demands are connected to one another: writing and speaking are *productive* activities while reading and listening are *receptive*. Additionally, reading and writing are "linked" to "written mathematical conventions," while listening and speaking encompass "oral language" (Aguirre & Bunch, 2012, p. 185). This multifaceted description of language aligns well with Moschkovich's (2015) definition of academic literacy, which positions the learner as an active participant in mathematical discourse. Moschkovich explains this phrasing by stating her desire to "shift from a focus on language as words to a broader sense of *literacy* as participation in practices and discourses" (p. 45).

When literacy is seen in the Vygotskian sense as "the understanding and communication of meaning" (Moll, 1992, p. 8), its relevance to mathematics education becomes even more apparent. After all, achieving learning with understanding is considered the "Holy Grail" of mathematics education (Hiebert & Carpenter, 1992, p. 65). If literacy within written and oral language is a core conduit through which mathematical ideas are communicated, then knowing how best to engage students in literacy practices is of upmost importance.

This report extends Aguirre and Bunch's and Moschkovich's exploration of language and literacy in mathematics by focusing on a curricular rather than instructional lens. The research questions considered are: (1) In what ways do mathematics curriculum materials present opportunities to address literacy demands of reading, writing, speaking, or listening? (2) In what ways do such curriculum materials connect opportunities for reading, writing, speaking, or listening?

Theoretical Framework

A contribution of this report is a framework which describes structured versus unstructured opportunities for reading, writing, speaking, and listening in mathematics. Wiggins (2001) notes how interdisciplinary curricular materials often position one discipline as subservient to the other, which results in an "approach [that] has little to do with teaching the concepts of [either discipline]" (p. 42). Genuine integration should instead promote shared concepts and processes present across the relevant disciplines, thus leading to better understanding of both disciplines (Wiggins, 2001).

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Shanahan and Shanahan (2008) discuss in particular how generalized literacy skills are insufficient within contents such as mathematics as students advance into secondary grades. Instead, teaching in these grades should emphasize disciplinary literacy skills unique to the specialized nature of the individual content (Shanahan & Shanahan, 2008). Curricular tasks which address literacy skills in generalized or subservient ways are considered *unstructured literacy opportunities* in this framework, while tasks which address mathematics-specific literacy skills (i.e. disciplinary literacy) are considered *structured literacy opportunities*. This framework expands beyond Shanahan and Shanahan's reading focus to consider Aguirre and Bunch's (2012) language demands of reading, writing, speaking, and listening. The disciplinary focus of each type of structured literacy opportunity is also described for each modality, as the mathematical purpose of curricular activities may vary. Such modality-specific descriptions of these opportunities follow.

Unstructured reading opportunities include any written mathematical text which students may read, while structured reading opportunities are tasks which specifically address elements of reading comprehension. This includes referencing surface comprehension of syntax and text features for students (Hoffer, 2012) or metacognitive modeling of deeper comprehension (Hoffer, 2012; Kenney, 2005). With such structures in mind, the role of vocabulary within these texts is also of interest. Certain words (called academic vocabulary) have more utility across domains while also being critical to comprehending mathematical text (Hoffer, 2012; Bay-Williams & Livers, 2009), so their inclusion in written tasks could better afford such opportunities. Thus, it is worth considering whether academic text is part of a structured reading opportunity or whether it is absent (which this framework describes as *simple text*).

Structured writing opportunities are found in tasks which require written explanations or justification, as well as tasks which promote refinement of mathematical ideas through written language (Hoffer, 2012; Kenney, 2005). Although many mathematical tasks use words/phrases that imply writing (e.g. "explain", "show how you know," etc.), this alone is unstructured. Since explicit guidance around when and how to write is critical to building writing skill in mathematics (Thompson, 2008), evidence of such opportunities is a prerequisite for structured writing. Additionally, such mathematical writing can serve different purposes. Writing can act as a *summative* assessment which provides better insight into student understanding (Miller, 1991), but it can also be used *formatively* throughout the problem-solving process to better develop students' metacognitive skills than oral communication alone (Pugalee, 2001).

Unstructured listening is a ubiquitous expectation of many mathematics classrooms (Aguirre & Bunch, 2012). Hintz and Tyson (2015) refer to "complex listening" as a structure to support this demand, where mathematics sense-making is encouraged in part by "*Directing* students towards what to listen for and whom to listen to" (p. 315, emphasis in original). While listening opportunities can serve to help students internalize information for themselves (e.g. understanding what the teacher is saying), they also can be used to help facilitate responses to others' ideas within discourse (Aguirre & Bunch, 2012).

Structure can be given to speaking opportunities by directing students toward cooperative speaking (Thompson, 2008) or exploratory talk (Mercer, Wegerif, & Dawes, 1999), where students are explicitly supported in understanding how to share time and space with their peers and collaborate respectfully. While such structures drive speaking in dialogically oriented math classrooms, classrooms which adhere to direct instruction emphasize students sharing their work individually in order to receive immediate, corrective feedback from the teacher (Munter, Stein, & Smith, 2015). This shows how speaking opportunities might be structured in collaborative *or* individualized ways depending on the preferred orientation to mathematical instruction.

Aguirre and Bunch (2012) detail how language modalities of reading, writing, speaking, and listening also connect with one another. The authors note that speaking and writing share productive

connections (ideas are produced) while reading and listening share receptive connections (ideas are received). Reading and writing are also both expressions of written language while speaking and listening are forms of oral language (Aguirre and Bunch, 2012). Thus, when multiple literacy connections appear in the same task, they can share connections related to student action (productive and receptive) or format (written and oral language).

Altogether these distinctions between unstructured and structured literacy opportunities, the varying types of structured opportunities, and the types of connections across language demands form the foundation of this report's framework. These are summarized in Figure 1.

Unstructured Literacy Opportunities (ULOs)	
• Unstructured Reading Opportunities (UR)	• Unstructured Listening Opportunities (UL)
• Unstructured Writing Opportunities (UW)	• Unstructured Speaking Opportunities (US)
Structured Literacy Opportunities (SLOs)	
• Structured Reading Opportunities (SR)	• Structured Speaking Opportunities (SS)
o Academic Text (SR-A)	o Cooperative Focus (SS-C)
o Simple Text (SR-S)	o Individualized Focus (SS-I)
• Structured Writing Opportunities (SW)	• Structured Listening Opportunities (SL)
o Formative Writing (SW-F)	o Listening to Respond (SL-R)
o Summative Writing (SW-S)	o Listening to Internalize (SL-I)
Literacy Connections	
• Written Language Connections (WLC) – reading and writing practices are both present in the task.	
• Oral Language Connections (OLC) – speaking and listening practices are both present in the task.	
• Receptive Connections (RC) – reading and listening practices are both present in the task.	
• Productive Connections (PC) – writing and speaking practices are both present in the task.	

Figure 1: Opportunities to Engage in Literacy Practices

Modes of Inquiry

A curricular analysis has been conducted to test the exploratory framework of this report. The Illustrative Mathematics (IM) curriculum has been chosen for this analysis because it is well-suited for the focus of this investigation. The IM curricular overview explicitly cites Aguirre and Bunch's 2012 chapter and claims that "embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math" (Illustrative Mathematics, 2019b, para. 1). This suggests that the "intended curriculum" (Herbel-Eisenmann, 2007, p. 344) of IM should at the very least align with Aguirre and Bunch's model and could also reveal how these ideas play out in the transition towards the written curriculum.

Specifically, this investigation will analyze the first 6 lessons of *Unit 2: Introducing Proportional Relationship* from the 7th grade IM curriculum materials. These lessons are chosen because they emphasize multiple representations of proportional relationships, and representation is seen as a conduit which supports enactment of literacy demands and drives holistic connections across literacy opportunities (Aguirre & Bunch, 2012). Since this unit is students' first introduction to the terminology of "proportional relationship" there are also opportunities for addressing academic vocabulary in the selected lessons.

Taken together, these different aspects of the unit afford ample opportunity for a preliminary analysis of literacy opportunities. The IM lessons are broken into distinct activities, so a level of analysis smaller than the lessons themselves is appropriate: different literacy demands can be addressed in the same lesson without necessarily being connected to one another if they arise in

discrete activities. Because IM activities generally break down into a "Launch" stage where a task is introduced and worked on and a "Synthesis" stage where student work or results are discussed, this study uses such task stages as the unit of analysis.

This approach created 40 task stages which were analyzed across the 6 lessons. A codebook was created which differentiated each type of opportunity and connection as summarized in the theoretical framework of this report. Reliability was established by enlisting a colleague to apply this codebook to the data set and checking for alignment with the author's results. Any initial disagreements in the data were discussed and resolved by the author and colleague. Each task stage was examined with the framework to determine whether any structured or unstructured literacy opportunities were described. When structured literacy opportunities (SLOs) were identified, they were also coded according to their appropriate sub-code (see Figure 1). The SLO results were then examined at the task level to determine if any tasks involved multiple SLOs. These tasks were coded as literacy connections as represented in Figure 1, because they provide students with opportunities to experience multiple related language modalities.

Results

The results of this analysis are presented in four main sections. The first two sections speak to the first research question regarding the ways that the Illustrative Mathematics (IM) curriculum materials address literacy opportunities of reading, writing, speaking, and listening. Results concerning structured literacy opportunities (SLOs) are examined in the first section, and these show that the curriculum materials employ multiple SLOs in every examined lesson but that productive opportunities for writing and speaking are more commonly found than receptive opportunities for reading and listening. The second section explores the unstructured literacy opportunities in these materials, and findings show that unstructured speaking and listening opportunities often appear together. The third section addresses the other research question regarding literacy connections and shows that the curriculum materials emphasize productive over receptive connections. The final section considers how the literacy practices were presented throughout the materials, noting how the way that such practices are phrased might promote an idea that SLOs should be reserved only for certain student populations.

Structured Literacy Opportunities

One overarching result of this analysis is that these curriculum materials employ structured literacy opportunities (SLOs) throughout their lessons, although with more focus on writing and speaking. Not only does every examined lesson include SLOs, but there is not a single lesson which does not employ at least 2 different SLOs. Additionally, all eight sub-codes for types of literacy opportunities were found, suggesting that these materials intentionally provide a range of opportunities to engage in literacy practices. As shown in Figure 2, 53 SLOs were coded across the tasks in these lessons. However, the different types of SLOs did not appear with equal frequency.

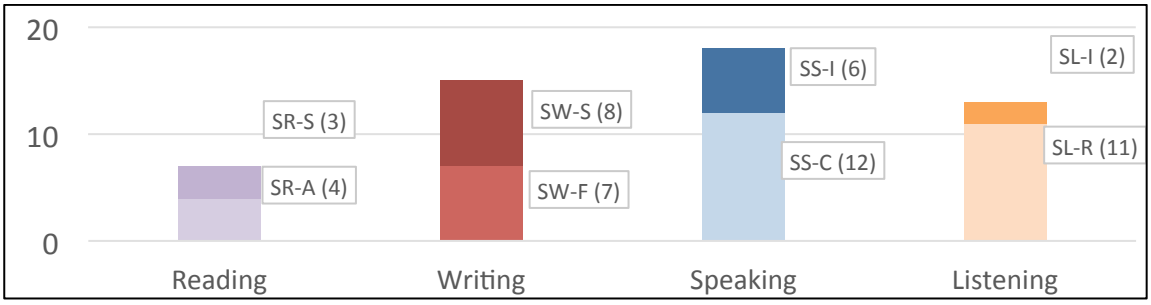

Figure 2: The frequency of each type of structured literacy opportunity with sub-codes

Structured opportunities for reading appear 7 times, writing 15 times, speaking 18 times, and listening 13 times. Despite every task being presented in the written language, reading comprehension is the least frequent structure seen in these tasks. Interestingly, listening SLOs (the other receptive modality besides reading) is the next least frequently referenced practice in these lessons. Given that most tasks in the IM curriculum materials consist of a "Launch" section with problems to complete (often with writing elements) and a "Synthesis" section grounded in discussion, it is not surprising to see high numbers of writing and speaking SLOs. What is surprising is that the receptive complements to these modalities are less frequently addressed. This becomes especially noted when looking at listening: Although there are 13 occurrences of such opportunities, 11 of these are *Listening to Respond* and 7 of those are paired with speaking SLOs. Figure 3 shows one such example from activity 5.2: The four sentence stems are structured speaking opportunities (cooperative focus) since they all orient students towards how to share time and space with their peers and collaborate respectfully (Thompson, 2008; Mercer, Wegerif, & Dawes, 1999), while "Why did you…?" and "I agree/disagree because…" are also structured listening opportunities (listening to respond) since they elevate complex listening skills (Hintz & Tyson, 2015) by directing students how to respond to specific arguments made by their partner. Given that these 7 paired routines blur the line between speaking and listening (since they emphasize discourse), this limits an explicit focus on listening. Such findings suggest that additional structures could be provided for reading and listening.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: "First, I _____ because ...." "I noticed _____ so I ...." "Why did you...?," "I agree/disagree because...."
Supports accessibility for: Language; Social-emotional skills

Figure 3: A paired speaking and listening opportunity (Illustrative Mathematics, 2019a)

Unstructured Literacy Opportunities

This analysis also shows that unstructured literacy opportunities (ULOs) exist within in the IM curriculum materials. Such ULOs appear throughout the analyzed lessons, with unstructured speaking and listening opportunities often being paired together. As seen in Figure, there are 40 unstructured literacy opportunities coded throughout these lessons, including 4 opportunities for reading, 2 for writing, 19 for speaking, and 15 for listening . These 40 ULOs number fewer than the 53 structured practices but still represent significant numbers.

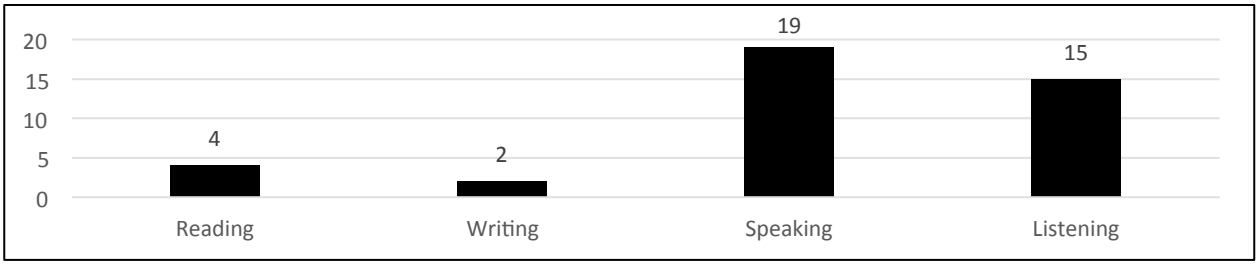

Figure 4: Frequency of unstructured literacy opportunities

24 of these instances arise from 12 pairs of listening and speaking ULOs. These unstructured pairs result from students being asked to compare or discuss with their peers (typically one partner) but without any oral language support structures (see Figure 5). Given that structured speaking and

listening opportunities *are* present throughout the IM curriculum, it is interesting that they are not consistently used when students are asked to engage in discourse. A more thorough analysis of the data could shed light on when structured speaking and listening opportunities are used versus when students are simply given unstructured speaking and listening opportunities.

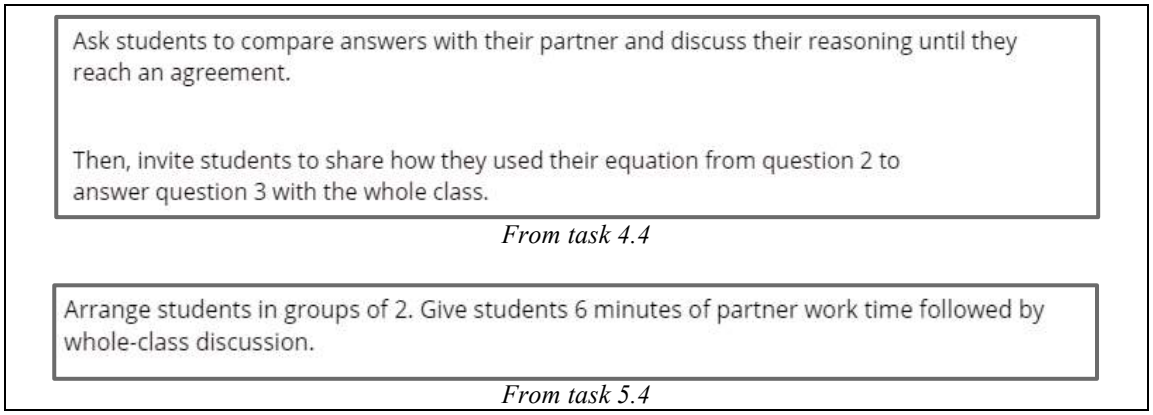

Figure 5: Unstructured speaking and listening opportunities (Illustrative Mathematics, 2019a)

Literacy Connections

The second research question for this report relates to the connections between literacy opportunities. Out of the 20 tasks explored, 14 of them had at least one connection (70% of all tasks) while 6 tasks showed two connections. Overall, 20 literacy connections were found. However, one finding of this analysis (shown in Figure 6) is that the types of literacy connections in the IM curriculum are not equally distributed. Productive connections and oral language connections both occur in 8 tasks, while written language connections only appear in 3 tasks and receptive connections do not appear at all. Additionally, 5 of the tasks include both productive and oral language connections, with each of these tasks having students write a response and then use that for discussion (see Figure 7 for such an example from activity 5.3). While this overall inconsistency is of note, the complete absence of receptive connections is especially a surprise. None of the analyzed tasks gave students structured opportunities to both read *and* listen. The limited number of written language connections (between reading and writing) further indicates that reading overall is an underutilized dimension of literacy in these materials.

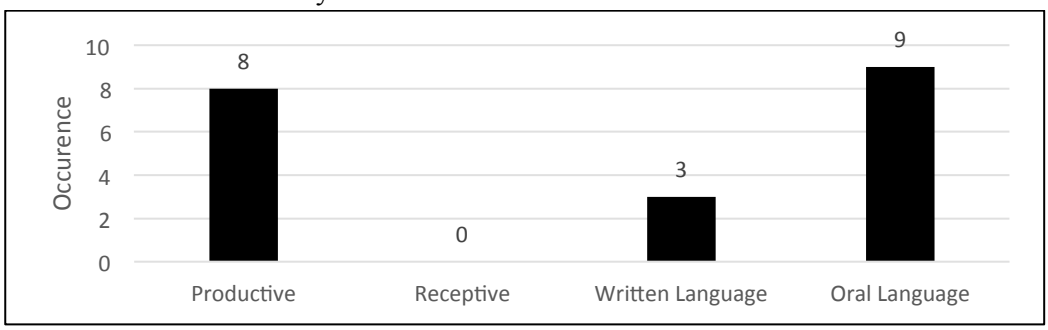

Figure 6: The number of each literacy connection that occurred in the selected IM lessons

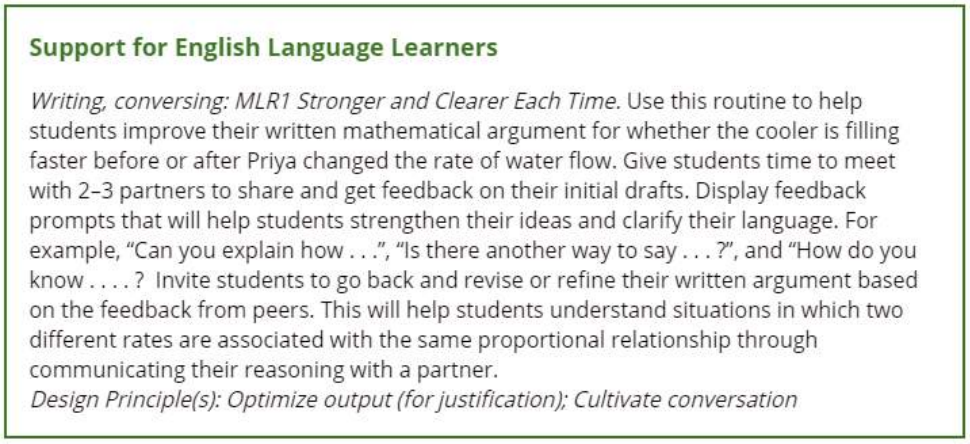

Figure 7: Productive and oral language connections within a task (Illustrative Mathematics, 2019a)

Framing of Literacy Practices in the IM Curriculum

Finally, an interesting trend that arose from the analysis is the way in which the IM curricular materials address literacy demands across different student populations. Illustrative Mathematics (2019) claims to integrate support for English Language Learners (ELLs) into their curriculum through what they call "Mathematical Language Routines" or MLRs (para. 22). As shown in Figure 7, this ELL priority is clearly established by bounding every MLR in a green border and titling it "Support for English Language Learners." Additionally, some structured literacy opportunities are included in bounded boxes titled "Support for Students with Disabilities," as shown in Figure 3. The MLRs ultimately account for 27 of the 53 structured literacy opportunities coded in this analysis, representing just over half of all such findings. The "Support for Students with Disabilities" directions account for another 4 opportunities. This means that such "bounded" curricular components account for a sizeable share of the total literacy opportunities found. All other opportunities, such as that shown in Figure 8, are embedded (unbounded) within the task launch or synthesis teacher guidance. This reveals two underlying conclusions: First, the MLRs that IL states are a core part of their curricular materials *do* appear in the analyzed tasks. Second, these routines are presented with a caveat – they are for ELLs and students with disabilities. While it is certainly true that language structures support ELL students (Aguirre and Bunch, 2012) and organizational structures aid low-performing students (Kenney, 2005, p. 45), the benefits of literacy opportunities are not limited to such groups. IM themselves admit as much when they state that "these instructional supports and practices (MLRs) can and should be used to support all students learning mathematics" (Illustrative Mathematics, 2019, para. 1). Despite this, the bounded nature of such supports and their frequent placement *after* all other teacher guidance within the curriculum materials could indicate exclusivity. Exploring how practitioners interpret structured literacy practices when they are built into the overall teacher-facing task instructions versus when they are separated is a consideration for future study.

Activity Synthesis	Select students to discuss and share their solutions. Ask them to identify difficulties which might include: getting started, noticing the pattern, dividing with decimals, completing the values in the table, creating the equation. This problem increases the level of difficulty by having so much missing information, and by using decimals in the table. It is important to identify if there are parts that are confusing for students to move them forward.

Figure 8: An embedded structured speaking opportunity from task 4.3 (Illustrative Mathematics, 2019a)

Discussion

This report provides numerous avenues for discussion for educators, curriculum designers, and researchers. First, this framework's descriptive language and distinction of different types of structured literacy opportunities can act as a roadmap for addressing such structures in mathematics curriculum materials. These definitions can also serve as a tool for strengthening otherwise unstructured literacy opportunities within curriculum materials or recognizing opportunities for connecting different modalities of literacy more consistently.

This framework can also better illuminate how curriculum materials are or are not considering multiple ways in which language relates to the learning of mathematics. Because each structured literacy opportunity has two distinct sub-codes, these allow for more nuanced discussion about the curriculum. For instance, the focus on speaking in the IM curriculum materials was usually cooperative rather than individual, and the focus on listening was largely to respond to peers' thinking rather than to internalize ideas for oneself. While this fits within the dialogic orientation to teaching, it limits opportunities aligned with the direct instruction model. Such an imbalance between the two sub-codes of speaking and two sub-codes of listening (but not an *absence* of any sub-code) gives rise to worthwhile questions about curriculum design and enactment: Are only the dialogically aligned opportunities desired? Should some curricular balance exist between cooperative versus individualized speaking, or listening to respond versus listening to internalize? This is an avenue for future research to consider.

Together, this framework and its application to IM curriculum materials provides insights into the role of language and literacy within mathematics. Applied only to a small sampling of materials, the framework illustrates the ways in which literacy opportunities are being structured and connected within tasks and demonstrates just how many literacy opportunities remain unstructured in these materials. Such results give clarity to the complex manifestations of literacy in the curriculum while also pointing towards further considerations which could advance our understanding of this critical aspect of learning mathematics.

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