WHAT MAKES A MATHEMATICS LESSON INTERESTING TO STUDENTS?

Leslie Dietiker
Boston University
dietiker@bu.edu

Rashmi Singh
Boston University
rsingh9@bu.edu

Meghan Riling
Boston University
mriling@bu.edu

Hector I. Nieves
Boston University
hnieves@bu.edu

How can we design mathematical lessons that spark student interest? To answer this, we analyzed teacher-designed and enacted lessons that students described as interesting for how the content unfolded. When compared to those the same students describe as uninteresting, multiple distinguishing characteristics are evident, such as the presence of misdirection, mathematical questions that remain unanswered for extended time, and a greater number of questions that are unanswered at each point of the lesson. Low-interest lessons did not contain many special narrative features and mostly had questions that were answered immediately. Our findings offer guidance for the design of lessons that can shift student mathematical dispositions.

Keywords: mathematics curriculum, narrative, aesthetic, mathematical story

What if mathematics lessons could be designed and enacted so they were as stimulating as *Harry Potter*, where students eagerly await the next installment? Literature is purposefully designed to capture and hold readers' attention; why not mathematical sequences? The ability to craft mathematical sequences that catch student attention and nurture a desire to continue to learn math would arguably have a positive impact on math education. That is, when a student becomes curious, they are more likely to engage with the content and increase attention, thereby increasing the potential for learning and deepening understanding (Csikszentmihalyi, 1990; Dewey, 1913; Guthrie, Hoa, Wigfield, Tonks, & Perencevich, 2005; Wong, 2007).

Despite major investment to improve mathematics curriculum, the content in most classrooms often sends the meta-message, *study this content because you need it to study related content later that you also will likely have little interest in or realize is even coming* (Chazan, 2000). Rather than provoking student imagination and curiosity in mathematics through sequencing curricular material, textbook authors rely on worldly contexts (conveying, *you should be interested in this because someday you might own a business and need to maximize profit*). Sinclair (2001) argues that this practice of relying on sources outside of mathematics to make mathematics interesting "endorses the belief that mathematics itself is an aesthetically sterile domain, or at least one whose potentialities are only realized through engagement with external domains of interest" (2001, p. 25). Drawing on Dewey's (1934) notion of *aesthetic* as a felt response to an experience rather than an attribute of an object, Sinclair asks, "Could we reverse the direction of the aesthetic flow, so that it originates in the mathematics?" (2001, p. 25).

Unfortunately, little is known about how decontextualized secondary school mathematics can be designed to be interesting and engaging. In this paper, we explore the characteristics of decontextualized secondary mathematics lessons that spur students' curiosity, captivate students with complex mathematical content, and compel students to engage and persevere, which we refer to as "mathematically captivating learning experiences" (MCLEs). The purpose of this paper is to address: *What characteristics, if any, distinguish high school mathematics lessons that students identify as interesting from those they describe as not interesting?*

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Theoretical Framework

In order to identify differences in how the mathematical ideas emerge and change as the lesson unfolds, this study interprets mathematical sequences that connect a beginning with an ending as a *mathematical story* (Dietiker, 2013, 2015b). Built from Bal's (2009) narratological framework, this interpretation foregrounds how mathematical characters are acted upon through mathematical action in mathematical settings. For example, mathematical characters are the mathematical objects brought into existence (objectified) through reference in the story, such as a function. Mathematical action describes the work of an actor (such as a student or teacher) in changing the mathematical ideas or objects of study, such as composing two functions to create a new function. Mathematical characters and actions are brought into being in a constructed "space" such as a white board or a coordinate plane, referred to as the mathematical setting.

For this study, a particularly important quality of a mathematical story is its *mathematical plot*; that is, the way it captivates and holds the interest of its audience. When a mathematical story hints of a future revelation, it may spur the formulation and pursuit of questions ("Why did the composition just end up with *x*?"), similar to how a reader of a literary story might wonder how the story will progress and continue reading. Thus, the mathematical plot describes the dynamically changing tension between what is already known and desired to be known by the participants as the story progresses (Dietiker, 2015b). It enables the description of how a mathematical sequence can generate suspense (by setting up anticipation for a result) and surprise (by revealing a different result than the one anticipated). Questions may span the entire story or may represent brief puzzles or mysteries. The progress made on each question, from when it is asked, to how students' understanding of it changes, to how it is abandoned or answered, constitutes a *story arc*. Since a mathematical story may involve answering multiple questions at any point along a sequence, multiple story arcs may arise over the course of the lesson and overlap at different intervals. The changing number of questions under pursuit by students throughout the lesson can be referred to as its *density* of inquiry.

Methods

This study identifies the distinguishing characteristics of interesting lessons that were designed and taught in the first of three design research cycles (Cobb, Stephan, McClain, & Gravemeijer, 2001; Edelson, 2002). The larger research project is an exploration of whether and how designing high school lessons as mathematical stories impact the aesthetic experiences of students. Six high school teachers, each with at least 4 years of experience, from three high schools with different curriculum and diverse demographic settings in the Northeastern region of the USA were recruited to participate in this study. The teachers worked in pairs, along with researchers, to design MCLEs for one or more of their classes. The participating courses, selected by teachers, spanned entry-level (e.g., Integrated Math 1) to advanced-level (e.g., calculus) and included both honors and non-honors. To support the design of MCLEs, the teachers attended a two-week professional development during the summer of 2018 where they learned about the mathematical story framework (Dietiker, 2015a, 2015b, 2016; Ryan & Dietiker, 2018) and participated in analyzing the mathematical plot of one lesson enactment.

An analysis of the complete set of MCLEs from the 2018-2019 school year, when compared with non-MCLEs from the same teacher and classes, revealed that the MCLEs did impact student interest measures positively (Dietiker et al., 2019). Yet, if these positive student reports are connected with non-mathematical factors (i.e., mood of the teacher, point in the semester), then the mathematical stories of the interesting lessons should not be significantly distinguishable from those of less interesting lessons. Thus, we designed the present study to learn whether the unfolding content of those lessons that students describe as interesting have characteristics are qualitatively different from those that students indicate are not interesting.

All 32 lessons generated in the 2018-2019 school year were observed by multiple researchers using the same protocol so that students would not be able to infer whether some lessons were special or not. The lessons were filmed using three video-cameras placed strategically to capture the teacher, students' facial expressions in the whole class, and the progress of a focus group of students. In addition to a central microphone, audio recorders were placed around the classroom to capture student discourse and the teacher wore a lapel mic. Immediately following each observed lesson, all participating students took a Lesson Experience Survey (LES) on their digital devices. In this survey, students were asked to rate their overall interest in the lesson on a scale of 1 to 4 and select three terms to describe their view of the lesson from 16 given descriptors, including negative, neutral, and positive options. More information about the design and testing of the LES can be found in Riling et al. (2019).

To recognize characteristics that distinguish high-interest lessons from low-interest lessons, the research team composed two groups of lessons by identifying each teacher's highest-interest and lowest-interest lessons based on students' LES responses. Pairing a low- and a high-interest lesson per class allowed both sets to include reports from the same students, rather than two groups of students who might have differing dispositions to mathematics to begin with. Only lessons with surveys from at least 10 students were included (this eliminated one lesson). We selected the lesson with the highest average interest measure for each teacher, using students' selection of positive descriptor to break ties. After selecting a teacher's high-interest lesson, the lesson for that class with the lowest average student interest level (with negative descriptors used to break ties) was selected for the low-interest group. Although the lessons were selected based on the interest level and student-selected descriptors from the LES, all lessons in the resulting "high interest group" were MCLEs and all lessons in the "low-interest group" were non-MCLEs.

To analyze the lessons in the high- and low-interest groups for their mathematical story characteristics, we first coded each for its mathematical plot. Then, we compared the mathematical plots of the two groups and identified characteristics that distinguished them. The plots relied on detailed transcripts for each lesson that included the discourse of a focal group, allowing us to note the progress those students made on the mathematical questions raised during groupwork and to include any questions the students asked while collaborating.

The transcripts were analyzed on three separate coding passes. On each pass, the research team coded separately in groups and then met to resolve differences. On the first pass, the team identified *acts* by tracking what mathematical characters, actions, and settings were in focus throughout the transcript and noting when these changed. On the second pass, the research team identified all mathematical questions that were raised, considered, and addressed throughout the lesson by a teacher, student, or some type of curriculum materials. Whereas some questions were recognized explicitly through verbal or written statements (e.g., "Find the root"), others were raised implicitly by images or situations experienced by students. Questions that were not mathematical (e.g., "Can I present my solution?") were not included.

On the final coding pass, the research team coded how what was known about each question changed across the acts of the lesson. For each question, the researchers used codes adapted from Barthes (1974) narrative theory to code contributions by teachers and students. These codes included foreshadowing of a question ("*proposal,*" marked with a 0 in the plot diagrams), when a question was raised ("*question,*" 1 when raised by a teacher and 2 when raised by a student), any explicit messages that the question would be answered ("*promise,*" 3), any progress made on answering the question ("*progress,*" 4 if made by a teacher, 5 if made by a student), and when, if ever, it was answered and thus closed ("*disclosure,*" marked with "D"). In addition to these codes, we also coded for interruptions to progress when the topic shifted so far from the question that it is no longer reasonable to assume the question may be addressed ("*suspension,*" marked with 9), or when there is a threat to progress toward an answer ("*jamming,*" marked with 8*)*. Finally, we coded any evidence of misdirection, in which lesson participants are misled in a consequential way. There are two types of misdirection: a *snare* (marked with 7) is an explicit error or lie, while an *equivocation* (marked with 6) is an encouragement to make a faulty assumption. To separately track who or what was responsible for a contribution, we also identified for each code whether the contribution came from the teacher, student, or environment (e.g., a worksheet).

These coding passes result in a comprehensive mapping of how participants within each lesson are moved to raise and answer questions, representing the mathematical plot of the lesson. For each question, the acts during which it is open (i.e., it is unanswered and it is reasonable to think that there is still a possibility of further progress on the question), along with all the codes for that question, form a story arc. The set of all story arcs describe how all of the mathematical content emerges and changes throughout the acts of the story.

Next, we qualitatively compared the mathematical plots and identified characteristics that appeared to distinguish the high- and low-interest lessons. We then compared quantitative dimensions of their mathematical plots such as number of acts, number of questions opened throughout the lesson, and story arc length. We compared the average number of coded questions per act, the average number of questions open and in progress per act ("mean density per act"), the proportion of story covered by these open questions ("mean arc length as proportion of story"), and the percentage of questions open for more than one act ("proportion of extended story arcs per total arcs"). A paired samples *t*-test was conducted to compare the mean differences between the two means of these measures for the two groups of lessons for $\alpha < 0.05$. A box plot diagram was made to compare the high- and low-interest lessons for measures.

Findings

In this section, we illustrate the differences between high-interest and low-interest lessons by describing a pair of high- and low-interest mathematical stories taught to one group of students by the same teacher. These mathematical stories are presented in present tense in the sequence in which events unfolded to highlight how what was known changed through the lesson. Next, we introduce contrasting characteristics of the stories' mathematical plots. Finally, we describe general patterns of high- and low-interest lessons, identifying characteristics of lessons that students find interesting.

This pair of lessons was selected because each contains many characteristics common to the other high- or low-interest lessons. These lessons were taught in an Algebra 2 Honors course with sophomores and juniors by Ms. Elm (pseudonym). The class had 28 students, 25 of whom participated in the study.

Low-Interest Mathematical Story

The class begins (Acts 1 through 3) with students working in partners on a "Do Now," which asks students to make sense of a newly defined operation ($a\&b = 3a - b$), including determining whether it is commutative and what its domain is. After Ms. Elm collects their work and verifies one of the answers, she reviews general principles of commutativity and domain.

In Act 4, Ms. Elm distributes a handout with questions about a range of topics, including domain, range, and percentages. The questions are multiple choice practice tasks for standardized tests. The focal group spends the rest of Act 4 working on a question about domain. In Acts 5 and 6, the group discusses a question about the range of a different function. In Act 7, they briefly complete more tasks about algebraic equations and operations.

In Acts 8 through 11, the focal group shifts to work on questions about percents. Each question is about a different aspect of percents and based in a different context: the reduced price of clothing item, what percentage one number is of another, and how much a company's profit increased.

Students engage with each other on each problem, although there is little evidence that they are challenged. The teacher concludes class by stating "we're gonna stop there for today."

High-Interest Mathematical Story

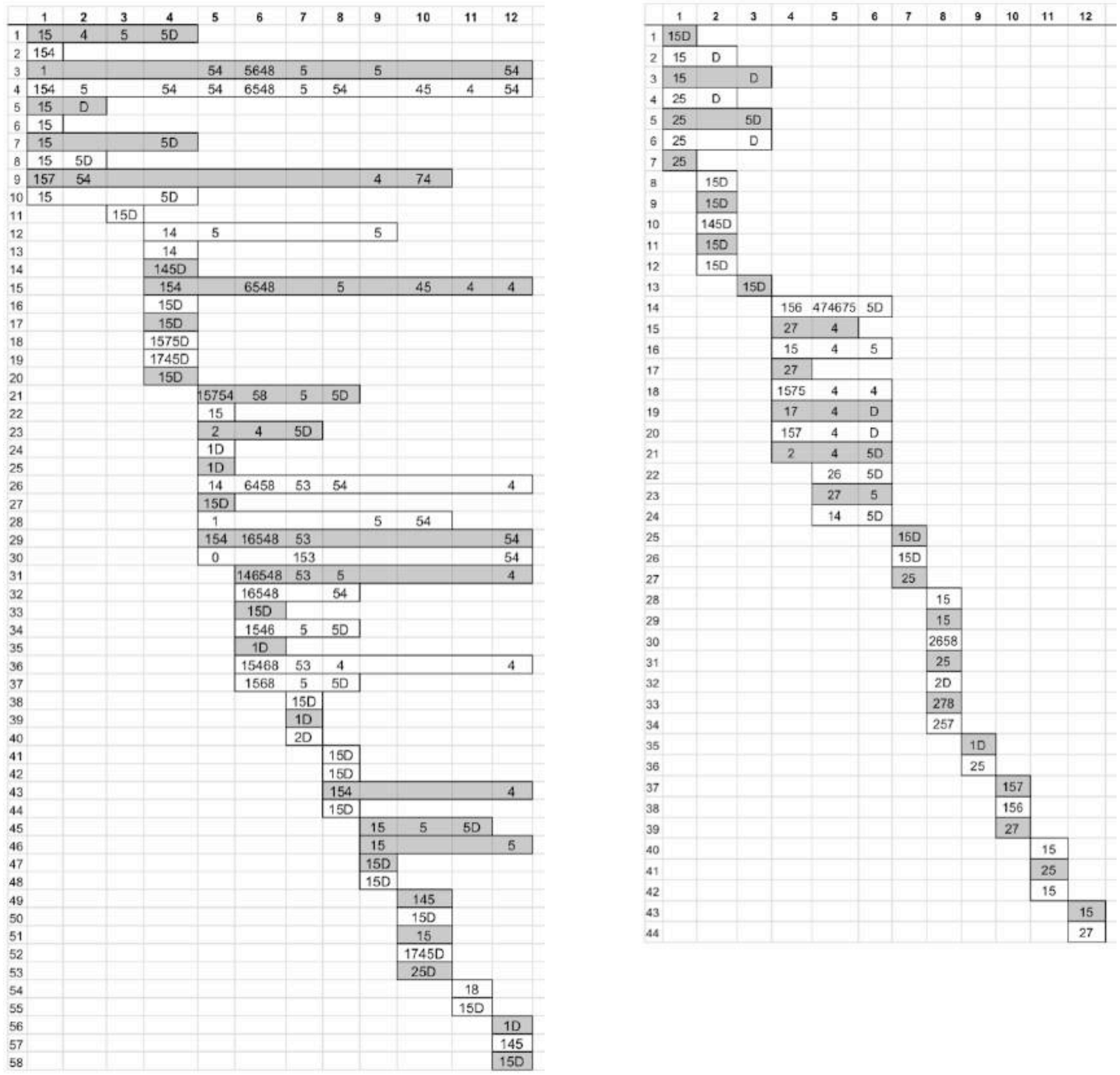
In Act 1, Ms. Elm explains that the lesson objective is to develop a strategy to identify the roots of polynomial functions. After displaying $f(x) = x^3 - 5x^2 - x + 5$ on the board and assigns each pair of students a value between –9 and 9, she challenges students to figure out whether they are "guilty as a root." First, she asks students to predict whether their value is a root and brainstorm ways to verify their guess. The focal group, which has the value 7, predicts that they cannot be a root because "if you do factoring by grouping, you wouldn't get 7 at all." In Act 2, Ms. Elm reviews synthetic division. In Acts 3 and 4, each group checks whether they are a root using synthetic division. The group whose value is 5 thinks they might be a root, which is confirmed. The focal group celebrates with a little dance. Next, the class finds the remaining roots (i.e. 1 and -1) by factoring the polynomial.

Students next test whether their group's value is a root of $f(x) = 2x^3 + x^2 - 16x - 15$. In Act 5, a student says that 8 is likely a root, explaining, "this is gonna sound kinda weird, but because of the two and the sixteen." This is the first time a student connects the polynomials' coefficients and roots. When it turns out that 8 is not a root, another group shares that 3 is a root. In Act 6, a student suggests that since 3 is a root, then -5 might be, because the y-intercept is -15. Ms. Elm responds by saying, "that's weird right, 'cause I actually really kind of agree here. That we need to do something to get ourselves to fifteen." She asks groups to use synthetic division to find the remaining roots. As students begin to do so, in Act 7, a student spontaneously claims that 2.5, not 5, is another root "because the leading coefficient, you need to divide that. It's two x minus five." Ms. Elm asks him to "hold that thought" and he reacts with contained excitement. In Act 8, the whole class finds the remaining quadratic once the cubic is divided by $x - 3$. Then, once the constant term is determined to be positive, students recognize that one of the remaining roots is not 2.5, but rather –2.5.

In Act 9, Ms. Elm elicits enthusiasm when she proposes a challenge: The class has four chances to identify the roots of $f(x) = 6x^4 + 35x^3 - 49x^2 + x + 7$. The first two choices, 6 and 7, are found to not be roots. A group proposes -7 as a root, and a student confirms it using synthetic division. In Acts 10 and 11, the class fully factors the polynomial and finds the remaining roots: 1, -⅓, and ½. In the last act, there is a growing sense during a class discussion that there is a multiplicative relationship between the coefficients and the roots. For homework, Ms. Elm asks students to reflect on how to identify polynomial roots.

The Comparison of the Mathematical Plots

These mathematical plots have several distinguishing characteristics, as can be observed in the mathematical plots of the high-interest lesson (Figure 1a) and low-interest lesson (Figure 1b). There were considerably more questions opened throughout Ms. Elm's high-interest lesson (75) as compared with her low-interest lesson (44). Interestingly, the percentage of the story arcs that remained open for more than one act was similar in both the high-interest lesson (27 out of 75, or 36%) and the low-interest lesson (15 out of 44, or 34%). However, the average arc length of the high-interest lesson is 2.6 acts long, which constitutes 22% of the lesson and is one act longer than the average length questions remained unanswered in the low-interest lesson (1.6 acts, or 13% of the lesson).



(a) (b)

Figure 1: The Mathematical Plots for Ms. Elm's (a) High-interest Lesson and (b) Low-interest Lesson. Each row represents a story arc. See methods section for code references.

In addition, most story arcs in the high-interest lesson contain acts in which no change in what is known about the question occurs. In almost a quarter of the story arcs (18 of 75), there is at least one act with no codes at all, providing the opportunity for students to build curiosity. This also provides a sense that not all questions that are raised will be answered immediately. In contrast, in the low-interest lesson, only three questions (#3, 5, and 6) have an act during which the question is open and yet no codes appear. For all other questions (41 of 44, or 93%), every act in which the question is open contains some change.

Stark differences are also evident in the density of these lessons (see Figure 3). Since the two lessons have a different number of acts, density is graphed across the percentage of the lesson that has passed. In the high-interest lesson, the density generally increases and then remains high, providing a lasting sense of mystery. Act 1 has 10 open questions, and by Act 7, almost twice as many questions are open (19). The number of open questions remains relatively high through the end of the lesson, as students continue to pursue their ideas about how to identify the roots of a given polynomial function. There is variation in the density of both lessons, as the tension alternately increases and decreases. Yet in the high-interest lesson, this variation does not return to the initial lower level, whereas the density in the low-interest lesson remains low with temporary dips. Overall, the density in the high-interest lesson was an average of 14.25 questions per act. In comparison, the low-interest lesson had a maximum density of 10 questions and an average density of 5.7 questions per act.

Figure 2: The Density of Ms. Elm's High-interest (red) and Low-interest (blue) Lessons.

Lastly, we found differences in the occurrence of special mathematical plot codes that represent interruptions and misdirection. The mathematical plot of the high-interest lesson has more instances of *jamming* in comparison with the low-interest lesson (11 vs. 2, respectively). Additionally, the high-interest lesson has a *proposal*, offering a sense of mystery, and five instances of *promise*, offering anticipation for an answer to come, whereas low-interest lesson has neither. In addition, the high-interest lesson has twice as many *equivocations* than low-interest lesson (10 vs. 5), while the latter has twice more (12 vs. 6) *snares*. Interestingly, the high-interest lesson had more misdirection from the teacher, both in terms of *equivocations* and *snares*, than the low interest lesson (3 vs. 1 and 1 vs. 0, respectively).

Characteristics that Distinguish High-interest and Low-interest Lessons

Across all 12 lessons, we identified multiple characteristics that distinguished high- and low-interest lessons significantly. Figure 4 shows the comparative measures for high- and low-interest lessons after the measures were standardized (i.e., mean = 0, vertical axis indicates the number of standard deviations from the mean). The questions in high-interest lessons remained unanswered for significantly more acts, as shown by a higher proportion of story arcs that lasted for more than one act (what we refer to as "extended questions") ($t(5) = 3.16$, $p < 0.05$), a longer mean arc length ($t(5)= 2.85$, $p < 0.05$), and the average arc length spanning a longer portion of the lesson ("mean arc length as % of story"; $t(5)= 4.21$, $p < 0.01$). In addition, the average number of open questions per act ("mean density per act") for the high-interest lessons is significantly greater than that of the low-interest lessons ($t(5)= 3.93$, $p < 0.05$). Similarly, the average number of changes to what is known (i.e., codes) per question in high-interest lessons is significantly greater than that of the low-interest lessons ("mean total codes per question"; $t(5)= 2.96$, $p < 0.05$). We found that high- and low-interest lessons have a similar percentage of acts with codes and disclosed formulated questions. Additionally, although higher interest lessons have more formulated questions and acts than low interest lessons, this difference is not distinguishable.

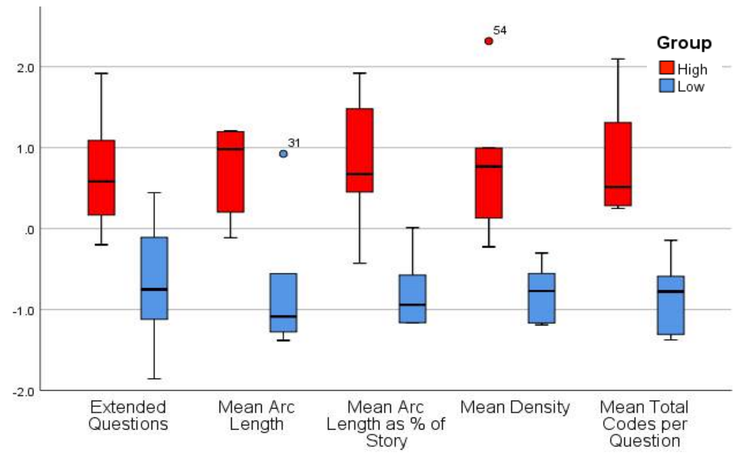


Figure 3: Comparison of Mathematical Plot Measures.

When comparing the frequency of the special mathematical plot codes by teachers (jamming, snare, equivocations, and promise), the high-interest lessons had a greater average frequency than the low-interest lessons. However, these differences were not statistically significant.

Discussion

We started this paper wondering how we can design mathematics lessons that compel students to become curious or excited. The characteristics described in this paper begin to answer this question. For example, the higher percentage of questions that are open for larger proportions of a lesson provide students extended opportunities to mathematically wonder, build understanding, and thus enjoy underlying mathematical concepts. In contrast, when questions of low-interest lessons are answered almost immediately, we are concerned that students may not have enough time to think deeply about mathematical concepts or relationships. Being confused, without the benefit of curiosity or anticipation, may hinder joy.

Knowledge of the characteristics of high-interest lessons can support educators and curriculum developers who wish to design mathematically captivating lessons that can positively impact students' experiences. For example, teachers might decide to mindfully include equivocations, which were not present in any of the low-interest lessons in our data, in order to enable surprise during their lessons. Or curriculum designers may decide to encourage teachers to delay giving answers to students, in order to permit students to wonder and anticipate for longer periods of time. Further research will hopefully uncover additional features that can be used to design reliably engaging mathematical learning experiences.

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